

# Periodic driving in quantum systems

German Sinuco

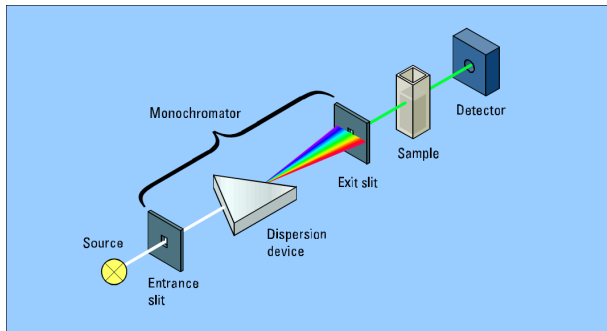
*Department of Physics and Astronomy, University of Sussex, Brighton, UK*

Imperial College London, 11<sup>th</sup> February 2015

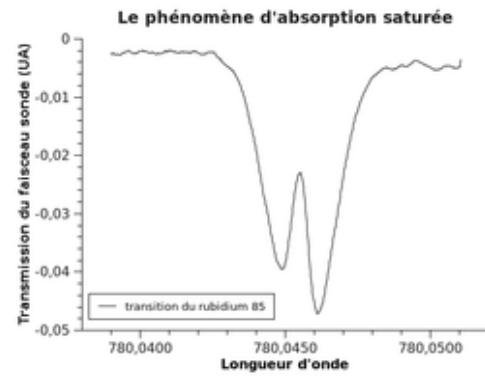
# Outline

- I. Context and this talk
- II. Floquet Theory
- III. Pumping in 1D systems
- IV. Dressed landscapes for cold atoms
- V. 2D Band engineering with periodic driving.
- VI. Conclusion

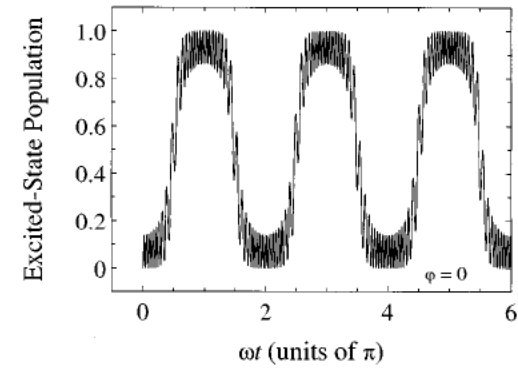
# I. Context and this talk



Absorption Spectroscopy



State population control and Rabi Oscillations

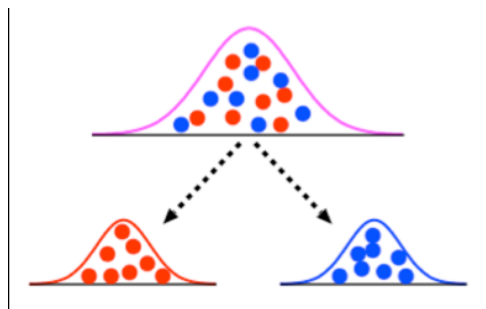


Garraway and Vitanov (1997)



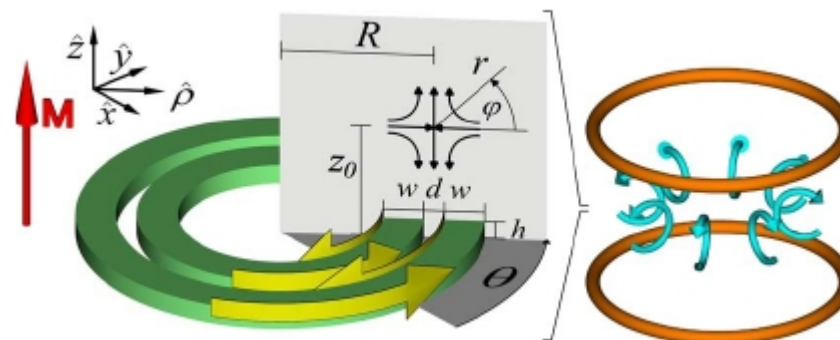
Nuclear Magnetic Resonance (NMR) and Magnetic Resonance Imaging (MRI)

## Control of quantum transport by periodic driving



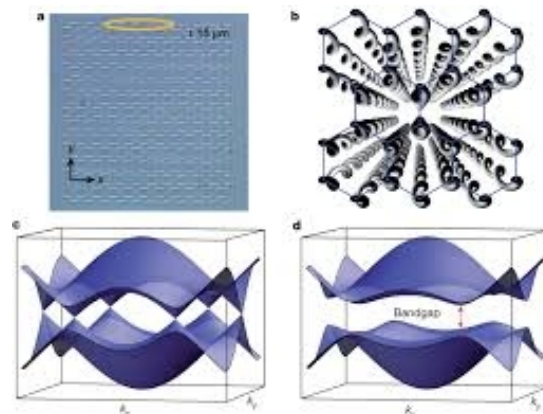
Website of Kazue Kudo, Ochanomizu University

## Tailoring of potential landscapes for cold atoms



Fernholz, *et al.*, PRA 75, 063406 (2007)

## Bloch-Band Engineering



Mikael C. Rechtsman, *et al.* Nature 496, 196 (2013)

This talk:

Single particle time-periodic Hamiltonian:

$$H(t) = H_0 + V(t)$$

$$H(t) = H(t+T)$$

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Single particle time-periodic Hamiltonian:

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Not in this talk:

- Decoherence and relaxation
- Many-body effects
- Transient dynamics

## II. Floquet theory



Jon Shirley, Phys. Rev. **138**, B 979 (1965)

Schrodinger Equation:  $i\hbar\partial_t|\Phi(t)\rangle=H(t)|\Phi(t)\rangle$   
 $H(t+T)=H(t)$

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Bloch Theorem in time domain:  $|\Phi_\alpha(t)\rangle=\exp(-i\frac{\epsilon_\alpha t}{\hbar})|\Psi_\alpha(t)\rangle$

$$|\Psi_\alpha(t)\rangle=|\Psi_\alpha(t+T)\rangle$$

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$$(H-i\hbar\partial_t)|\Psi_\alpha\rangle=\epsilon_\alpha|\Psi_\alpha\rangle$$

Fourier decomposition:  $\langle n|\Psi_\alpha(t)\rangle=\sum_q C_{\alpha,q}^n \exp(iq\omega t)$

$\{|n\rangle\}$ : complete basis of system Hilbert space

$$(H - i\hbar\partial_t)|\Psi_\alpha\rangle = \epsilon_\alpha|\Psi_\alpha\rangle$$

$$\langle n|\Psi_\alpha(t)\rangle = \sum_q C_{\alpha,q}^n \exp(iq\omega t)$$

$$H = H_0 + V(t)$$

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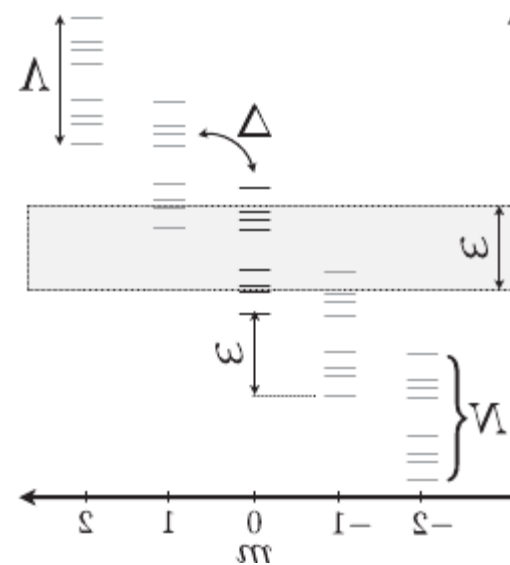
$$\langle n|\Psi_\alpha(t)\rangle = \sum_q C_{\alpha,q}^n \exp(iq\omega t)$$

$$H = H_0 + V(t)$$

(a)

$$\mathcal{H}(\mathbf{k}) = \begin{pmatrix} \dots & 1 & m' & -1 & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ H_0 + \omega & \Delta & & & 1 \\ \Delta^\dagger & H_0 & \Delta & & 0 \\ & & & & m \\ \Delta^\dagger & H_0 - \omega & & & -1 \\ \vdots & \vdots & \vdots & \vdots & \vdots \end{pmatrix}$$

Rudner, *et al.*, PRX 3, 031005 (2013)



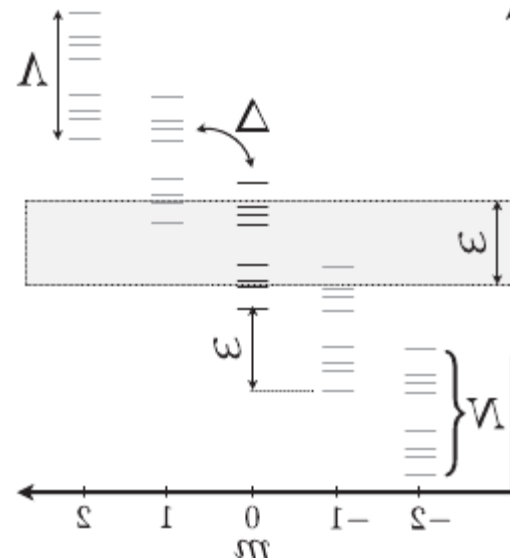
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Rudner, *et al.*, PRX 3, 031005 (2013)

$$H_F = H_0 \otimes 1 + 1 \otimes \hbar\omega \hat{n} + \sum_{n \neq 0} V_n \otimes \sigma_n$$

$$V(t) = \sum_{n \neq 0} V_n \exp(in\omega t) \quad \sigma_n |m\rangle = |m+n\rangle$$

## Floquet spectrum and the evolution operator

$$U(t_0 + T, t_0) |\Phi_\alpha(t_0)\rangle = |\Phi_\alpha(t_0 + T)\rangle$$



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$$H_{eff} = \sum \epsilon_\alpha |\Psi_\alpha\rangle \langle \Psi_\alpha|$$

$$U_F = U(t_0 + T, t_0) = \exp(-iH_{eff} T/\hbar)$$

Also:

$$i\hbar\partial_t|\Phi(t)\rangle=H(t)|\Phi(t)\rangle$$

The time-dependent problem can be also solved approximately by finding a unitary transformation,  $U(t)$ , such that the transformed Hamiltonian is dominated by a time-independent component:

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$$\bar{H}=U^\dagger H(t)U-i\hbar U^\dagger\partial_t U=H_0+\Delta H(t)$$

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- Rotating Wave Approximation (Ramsey, 1956).
- Pegg and Series (1973).
- Magnus Expansion.
- Hemerich (PRA, 2010), Poletti & Kollath (PRA, 2011)
- Goldman & Dalibard (arxiv:2014)
- Mintert (PRL,2013)

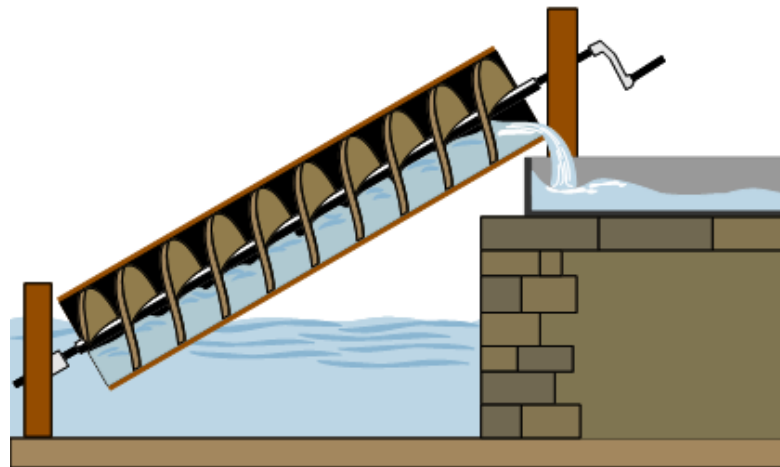
### III. Pumping in 1D systems

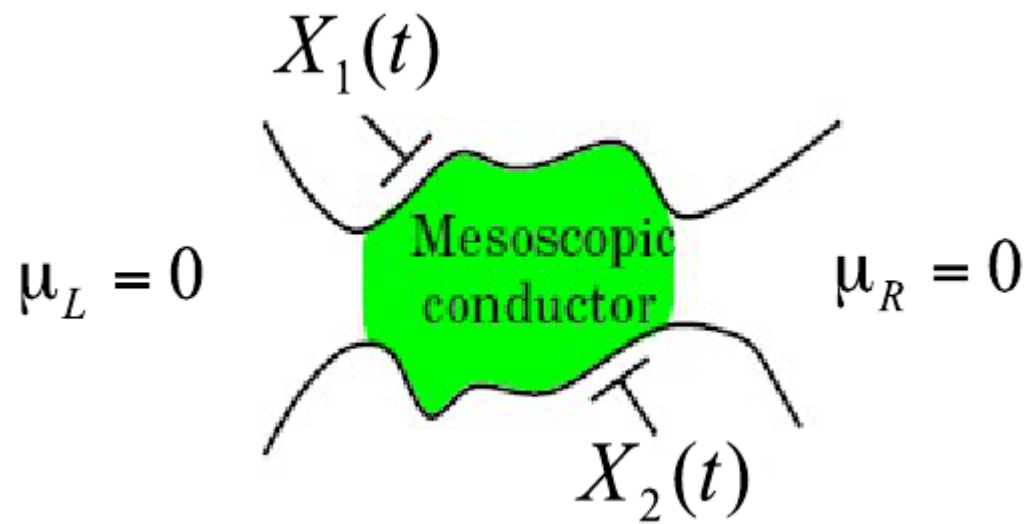
In a pump, transport of particles (or fluids) is generated following a periodic deformation of the system parameters.

Transport can occur even under the action of a bias field, e.g. gravity, pressure

Example:

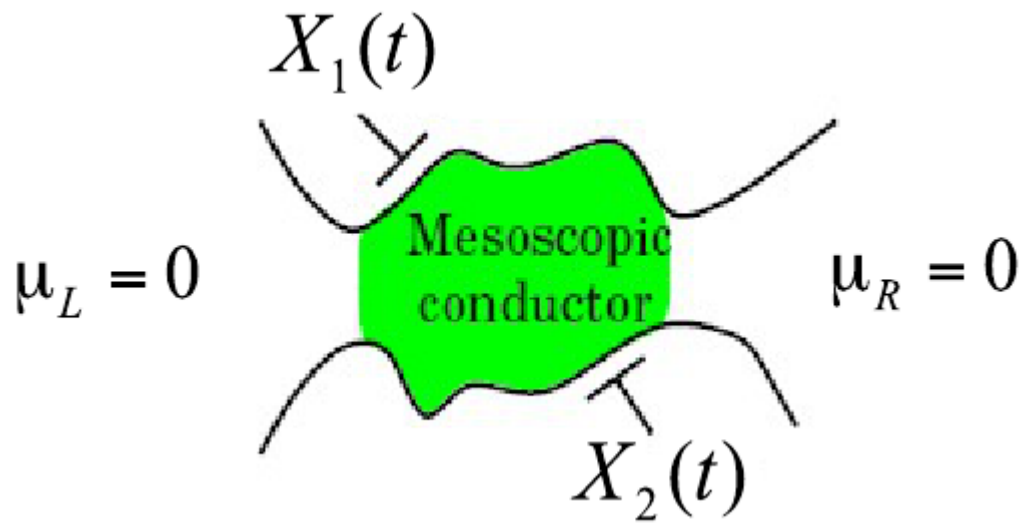
Archimedes' screw



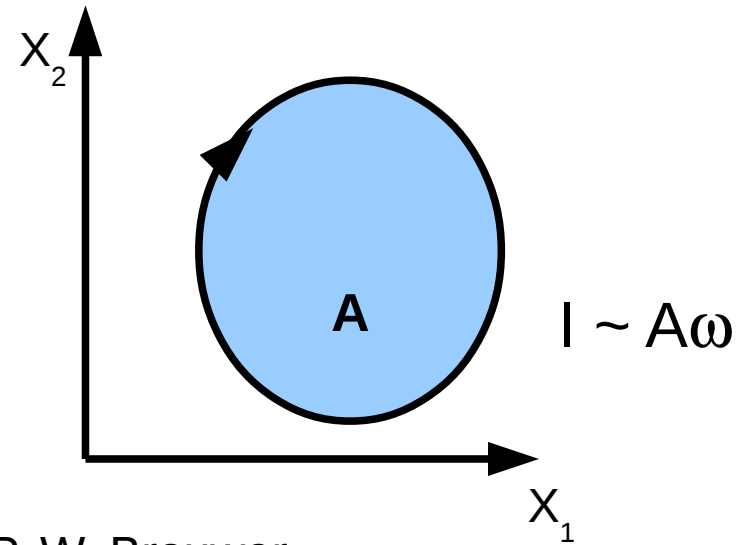


Spivak, *et al.*  
Phys. Rev. Lett **82**, 608 (1999)

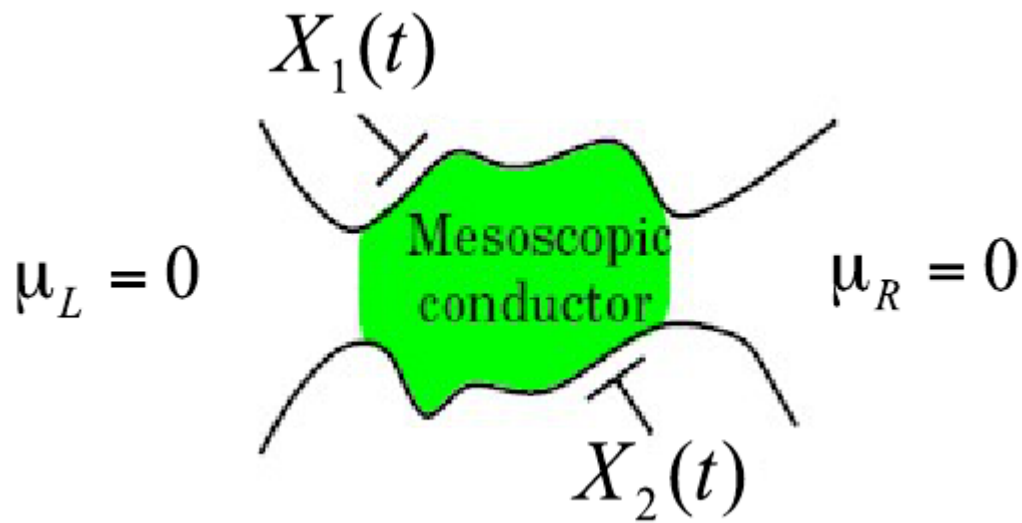




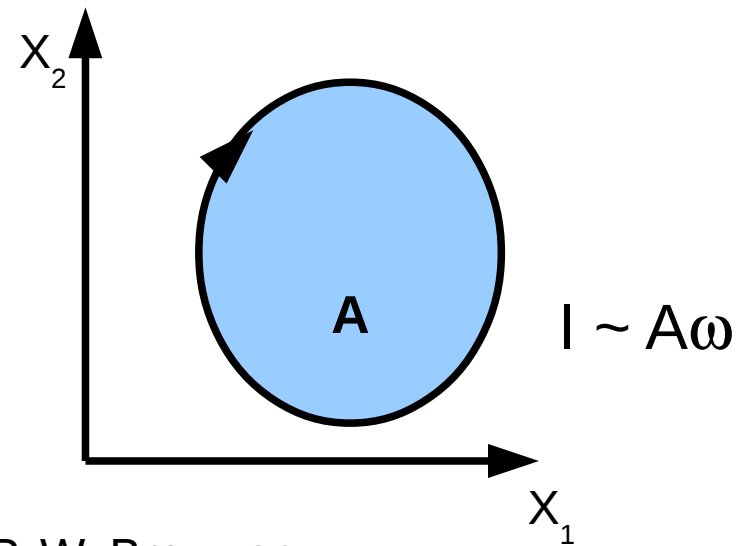
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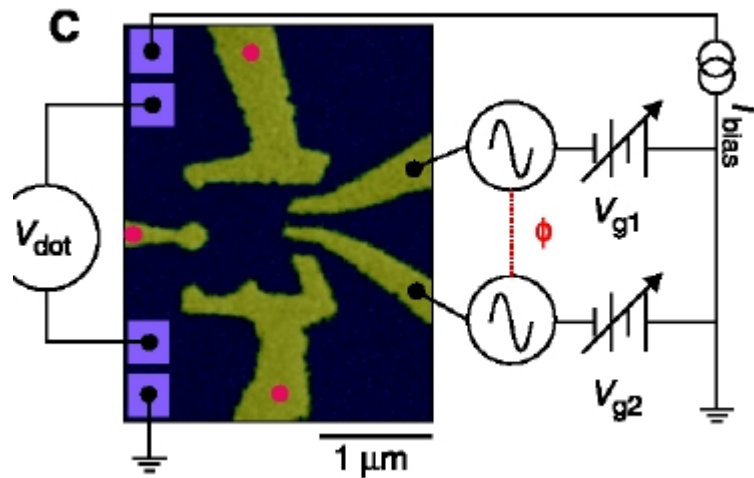
P. W. Brouwer ,  
 Phys. Rev. B **58**, 10135(R) (1998)



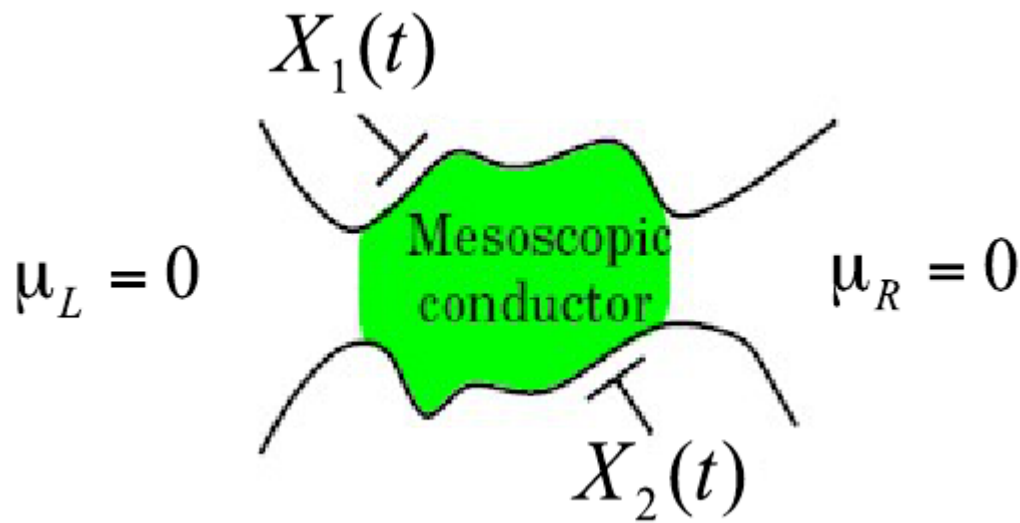
Spivak, *et al.*  
 Phys. Rev. Lett **82**, 608 (1999)



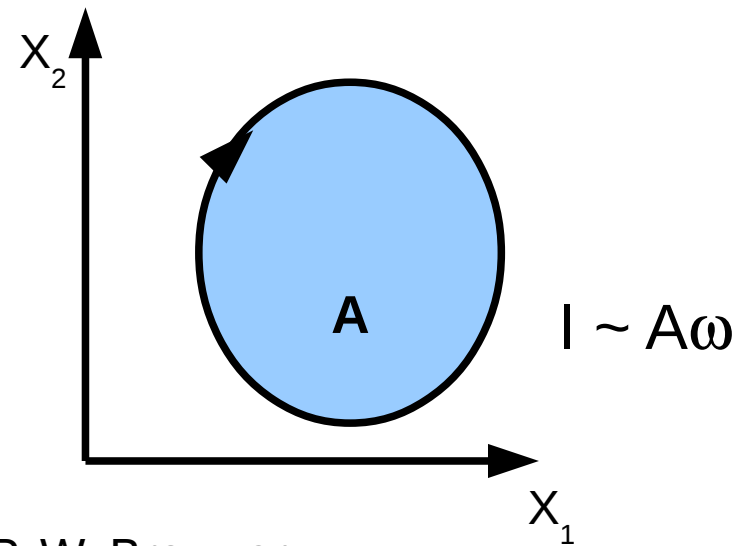
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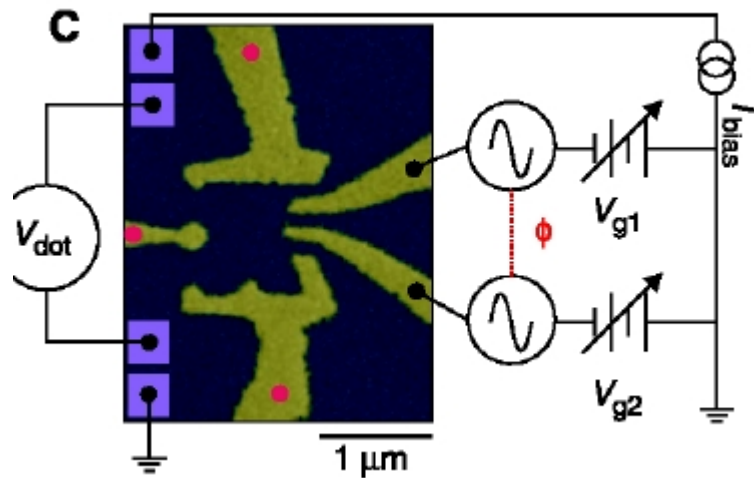
Swites, *et al.*, Science **83**, 1905 (1999)



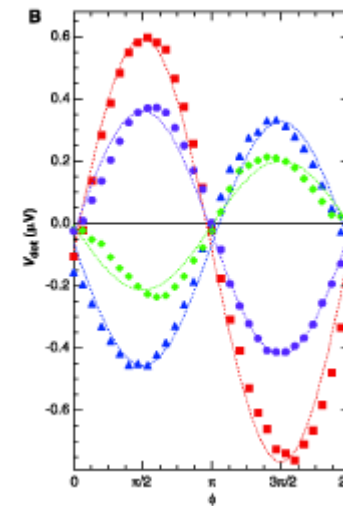
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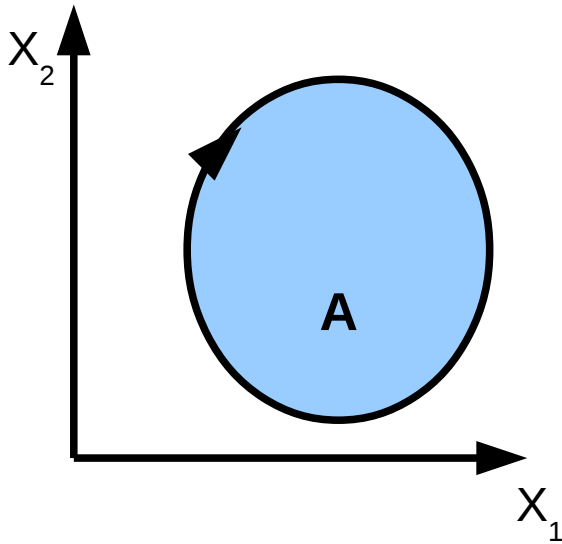


Brower, Buttiker and Avron (late 90's - early 2000's)

Analogue of the Landauer Formula: Current in terms of the scattering matrix  
Geometric description of charge transport in mesoscopic systems.

Brower, Buttiker and Avron (late 90's early 2000s)

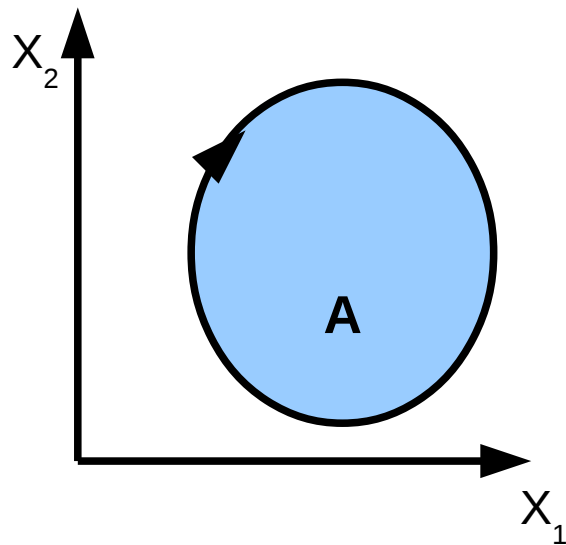
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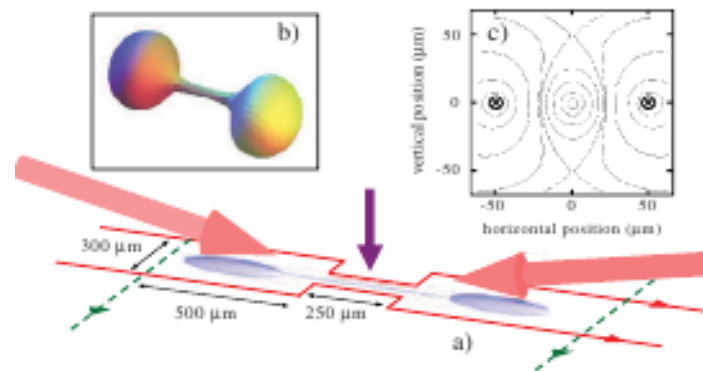
$$I \approx \frac{-i\omega e}{4\pi^2} \int_A dX_1 dX_2 [(\partial_{X_1} S) S^\dagger, (\partial_{X_2} S) S^\dagger]$$

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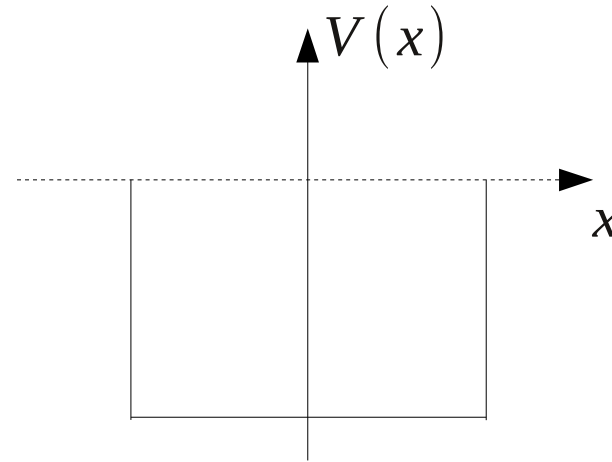
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Das and Aubin, PRL 103, 123007 (2009)

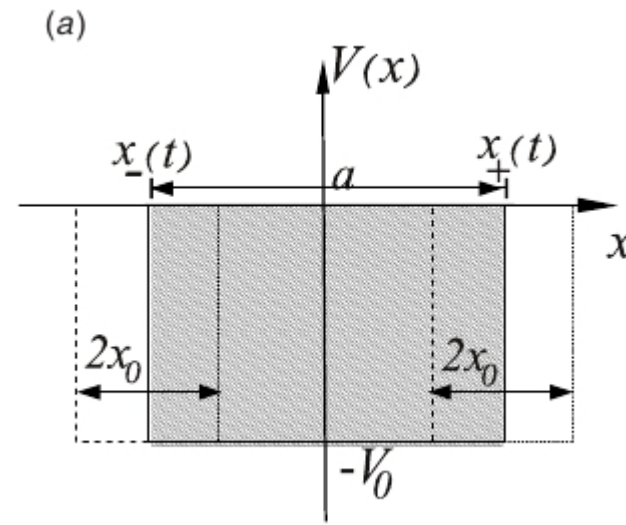
$$H = \frac{P^2}{2m} + V(x, t)$$

$$V(x, t) = V(x, t + T)$$



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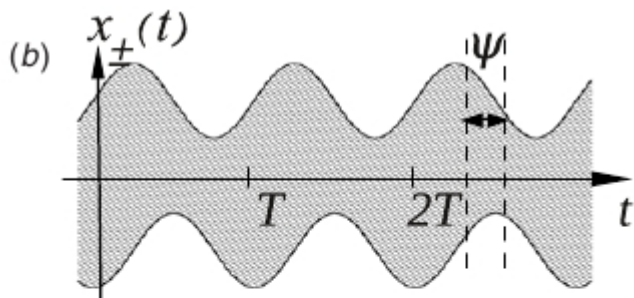
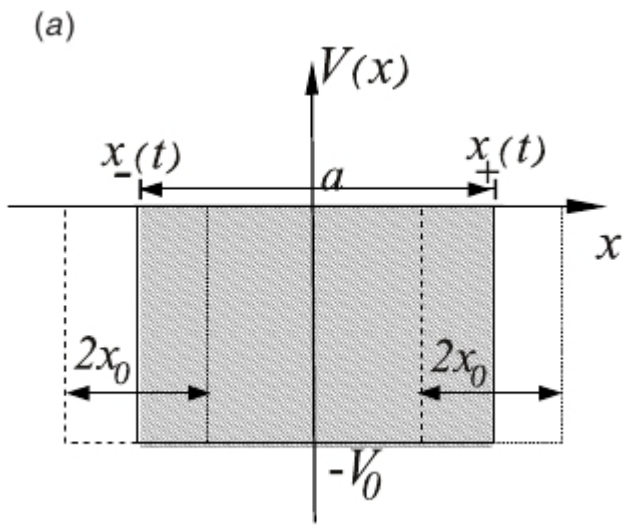
$$V(x, t) = V(x, t + T)$$



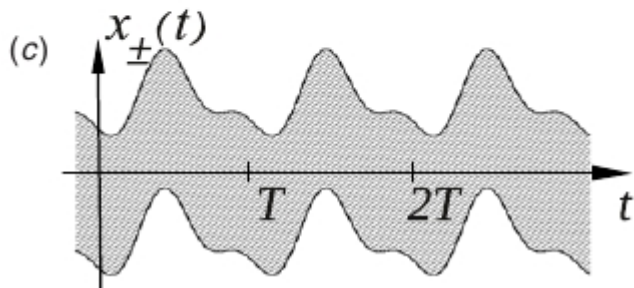


$$H = \frac{P^2}{2m} + V(x, t)$$

$$V(x, t) = V(x, t + T)$$



$$x(t) = \pm 1 + x_0 \cos(\omega t \pm \psi/2)$$



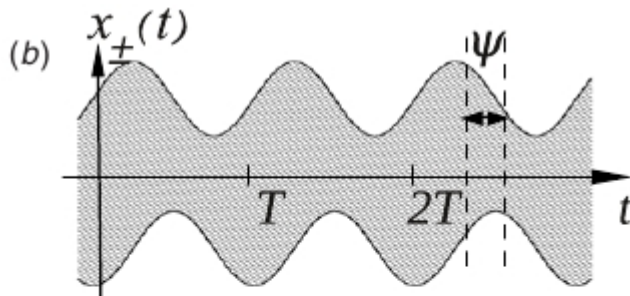
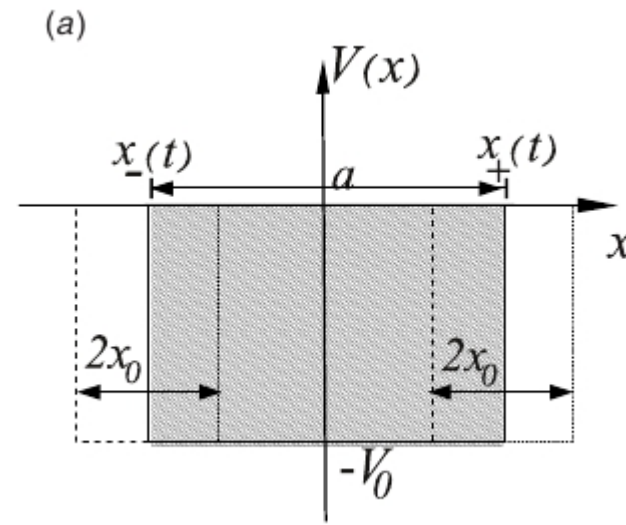
$$x(t) = x_0 f(t)$$

$$x(t) = x_0 (\cos(\omega t) + \gamma \cos(2\omega t - \varphi))$$

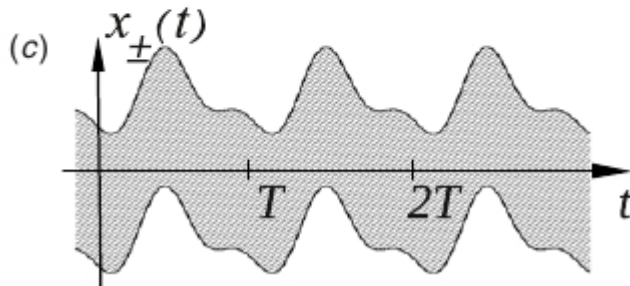
Castaneda, Dittrich and **GS**, JPA **45**, 395102 (2012)

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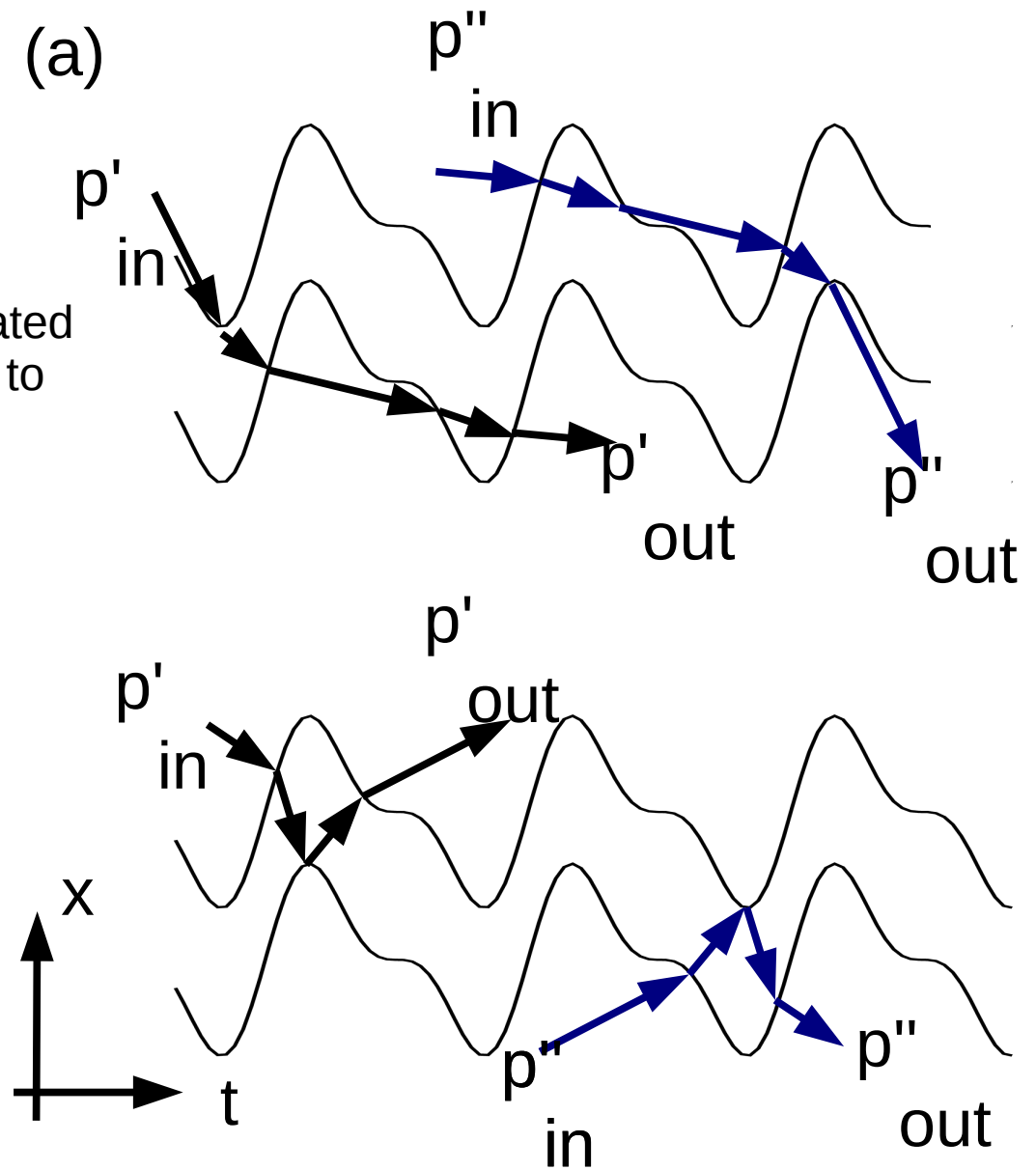
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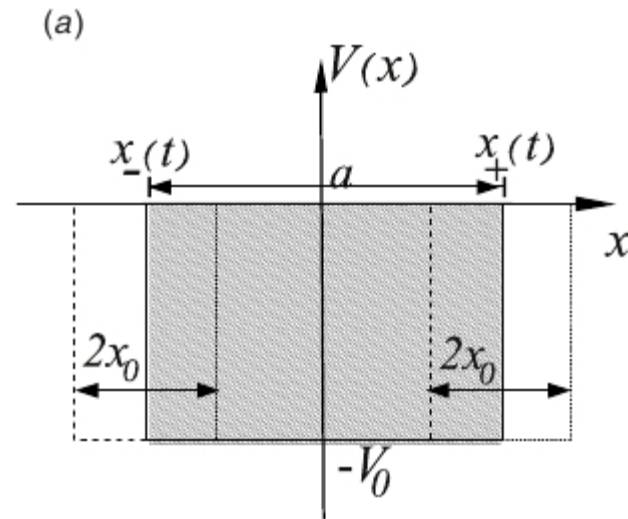
$$I(E) = \left\langle (R^{rr} + T^{lr} - R^{ll} - T^{rl}) \right\rangle_{\text{arrival time}}$$

To avoid systematic cancellation due to counter-propagating trajectory pairs related by spatial reflection symmetry, we have to break time-reversal invariance of the potential.

For the single-parameter driving this requires

$$f(-t + t_0) \neq \pm f(t)$$





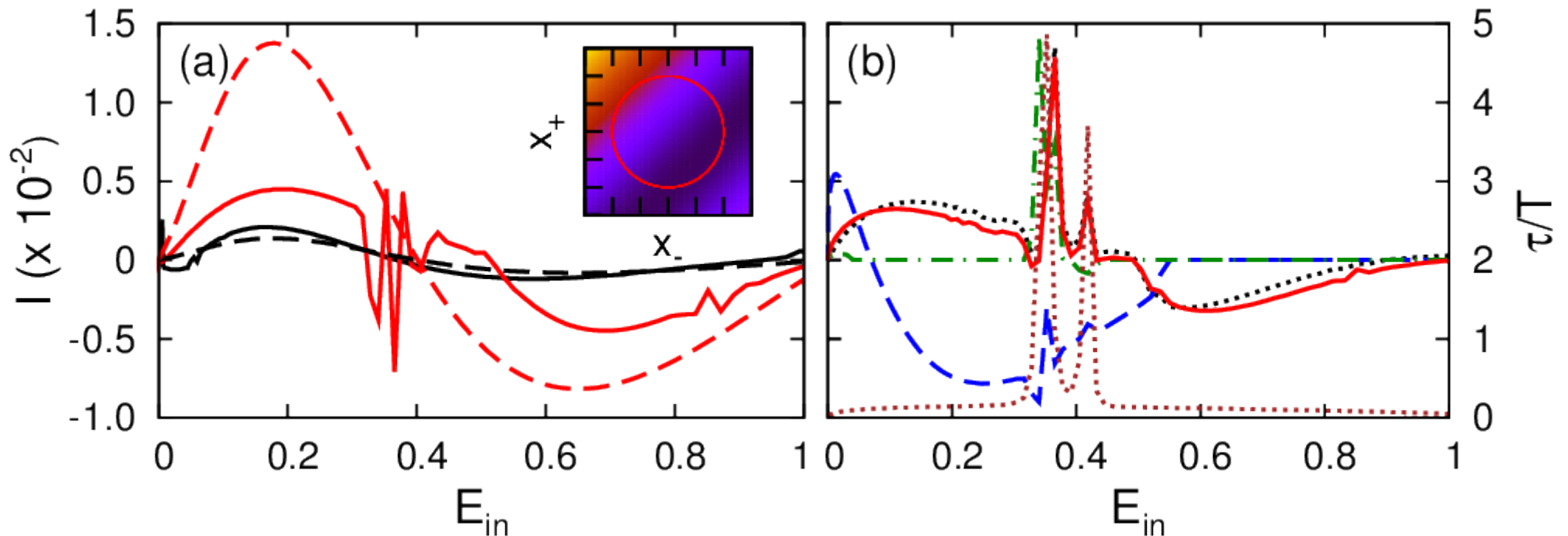
Transmission and reflection coefficients defined in terms of scattering matrices.

$$S_{0,n_{out}}^{\sigma,-\sigma} = \langle k_0(E_0 + n_{out} \hbar \omega) | (U_F)^N | k_0(E_0) \rangle$$

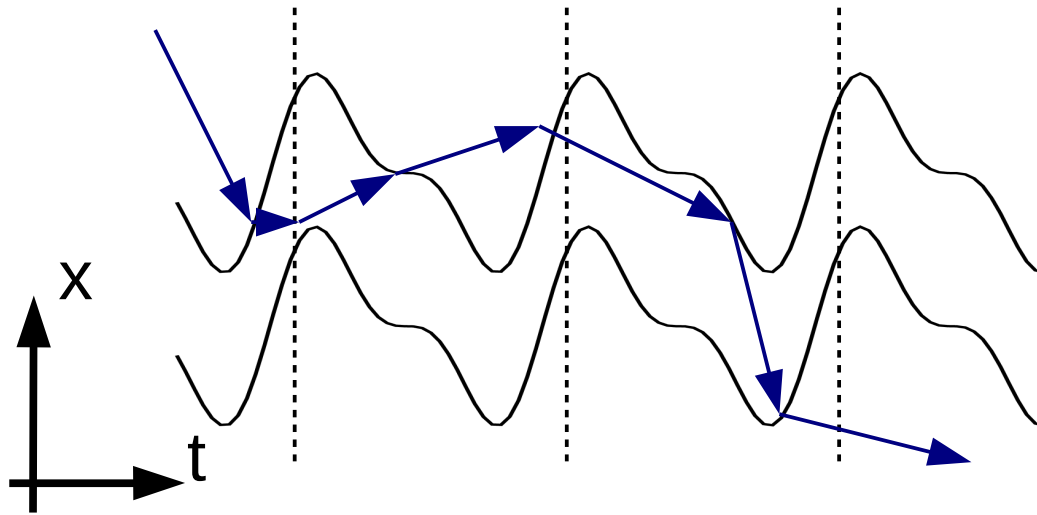
$$T^{\sigma,-\sigma}(E_0) = \sum_{n \neq 0} |S_{0,n}^{\sigma,-\sigma}(E_0)|^2$$

$$I(E_0) = (R^{rr} + T^{lr} - R^{ll} - T^{rl})$$

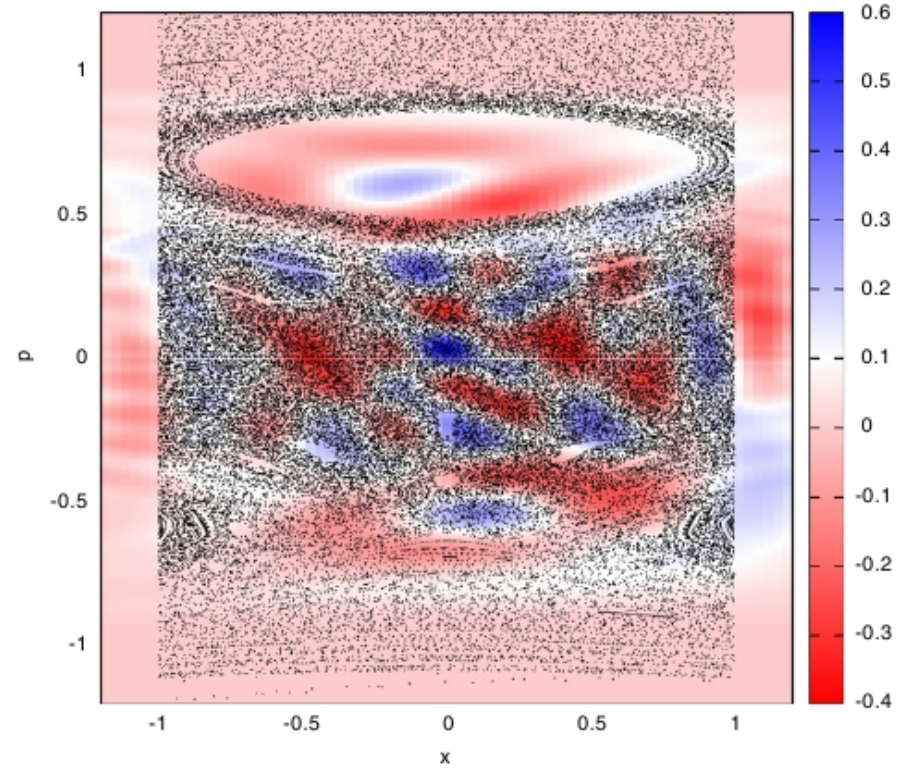
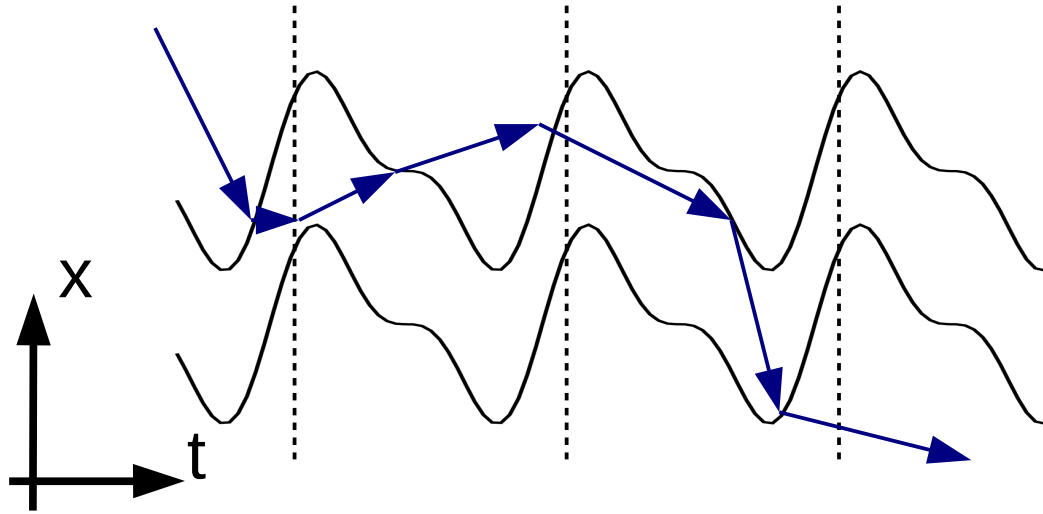
$$I(E_0) = (R^{rr} + T^{lr} - R^{ll} - T^{rl})$$



QM Current as function of the incoming energy. (a) Floquet (full) vs. adiabatic. (dashed) for slow (black) and fast (red) two parameter driving. (b) Total current for single-parameter driving (full red)



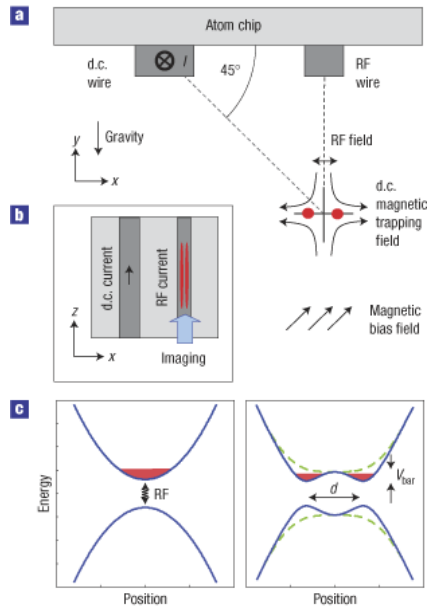
Quantum-Classical correspondence: Transport is a manifestation of the same underlying dynamical mechanism



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## IV. Dressed landscapes for cold atoms

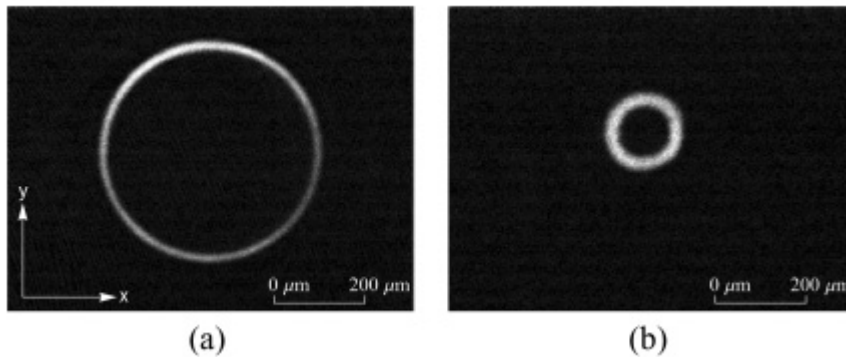




Atom-chip design of a dressed potential:

- Highly controllable configuration: complex landscapes using simple conductor layouts.
- Large trapping frequencies in close proximity to chip surface.
- Simultaneous trapping of two hyperfine states: Microwave coupling can be used for applications as in Optical lattices.

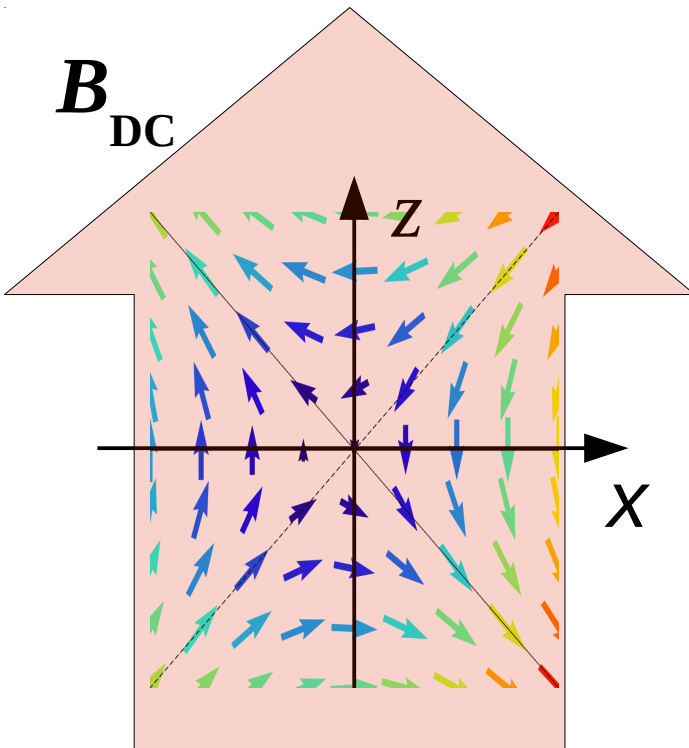
Matter-wave interferometry with RF-dressing  
(T. Schumm, Nat. Phys.(2005))



@ Oxford, PRA 83, 043408 (2011)

Consider an alkali atom slowly moving through a region with inhomogeneous static and AC fields:

$$H = \frac{P^2}{2m} + m_F g_F \mathbf{B}_{DC} \cdot \hat{\mathbf{F}} + m_F g_F \mathbf{B}_{AC} \cdot \hat{\mathbf{F}} \cos \omega t$$

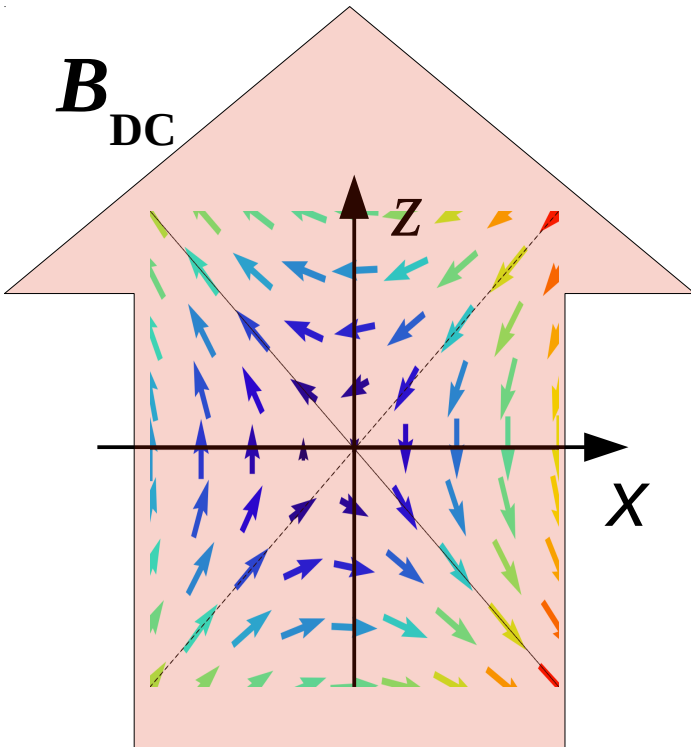


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Perform a local rotation of the axis, such that the static field is aligned with the z-axis. Then move to a rotating frame of reference

$$U(\mathbf{r}) = \exp(-i\omega t \hat{\mathbf{F}}_z) R_{DC}(\mathbf{r})$$



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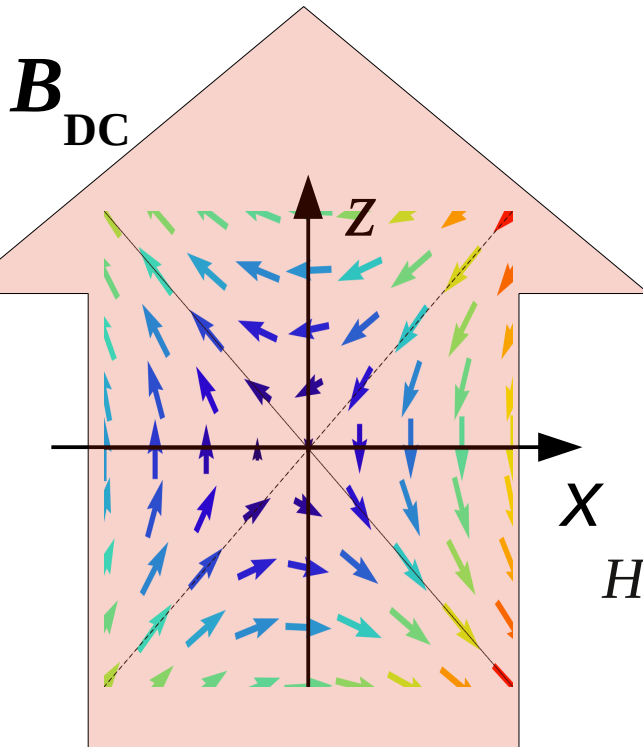
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$$U(\mathbf{r}) = \exp(-i\omega t \hat{\mathbf{F}}_z) R_{DC}(\mathbf{r})$$

$$H = \frac{(\mathbf{P} + \mathbf{A})^2}{2m} + (m_F g_F B_{DC} - \hbar \omega) \hat{\mathbf{F}}_z + \frac{m_F g_F B_{AC}}{2} \hat{\mathbf{F}}_x + .$$

$$\frac{m_F g_F B_{AC}}{2} (\hat{\mathbf{F}}_x \cos 2\omega t - \hat{\mathbf{F}}_y \sin 2\omega t) + m_F g_F B_{AC}^z \hat{\mathbf{F}}_z \cos \omega t$$

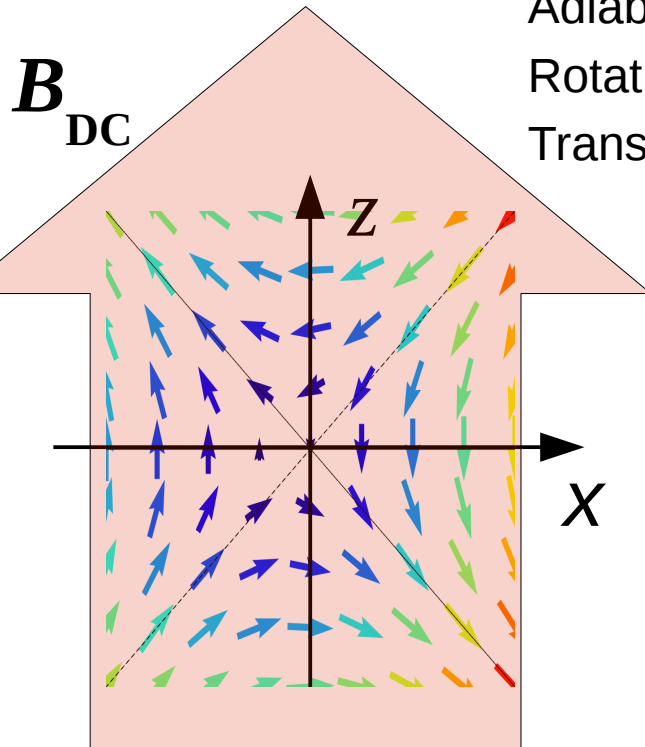
$$\mathbf{A}(\mathbf{r}) = -i\hbar U(\mathbf{r})^{-1} [\nabla U(\mathbf{r})]$$



Adiabatic approximation: Neglect the gauge field  $\mathbf{A}$ .

Rotating Wave Approximation (RWA): neglect the counter rotating term

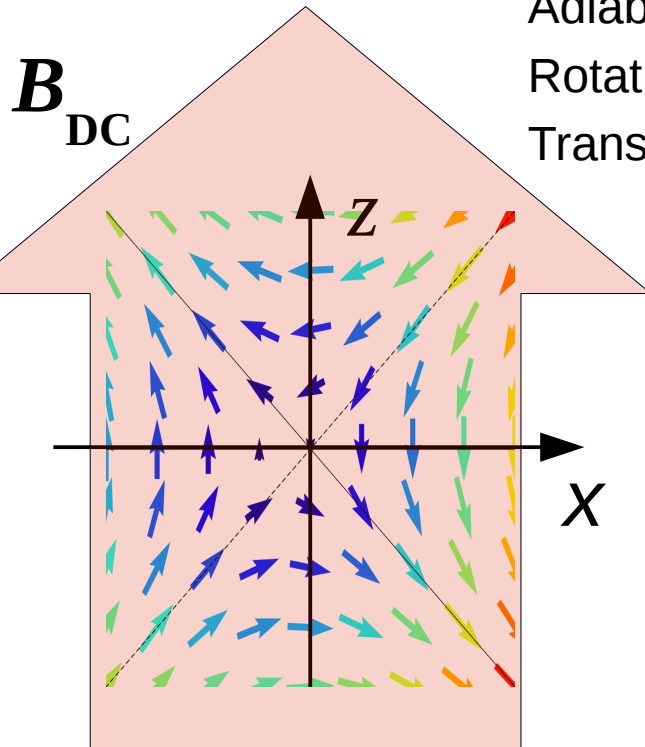
Transverse field only: neglect misaligned fields.



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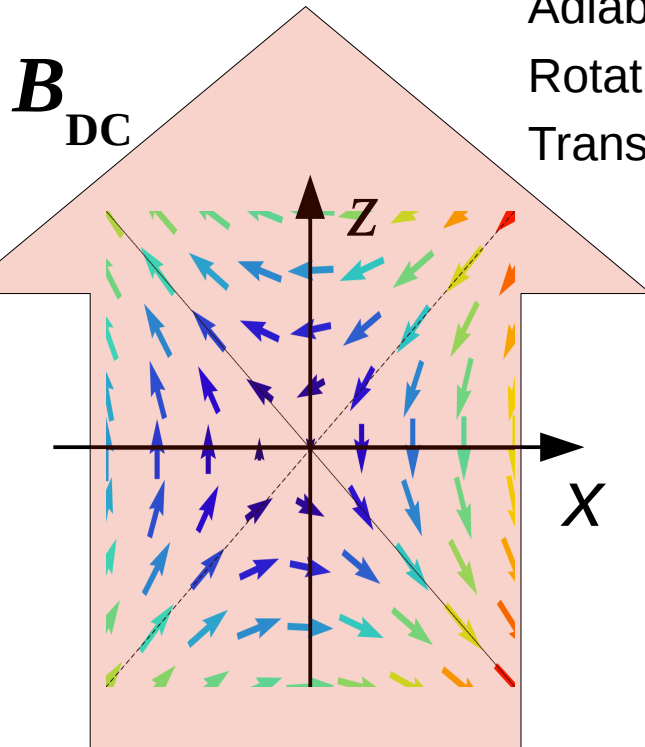


$$H \approx \frac{\mathbf{P}^2}{2m} + (m_F g_F B_{DC} - \hbar \omega) \hat{F}_z + \frac{m_F g_F B_{AC}}{2} \hat{F}_x$$

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Transverse field only: neglect misaligned fields.



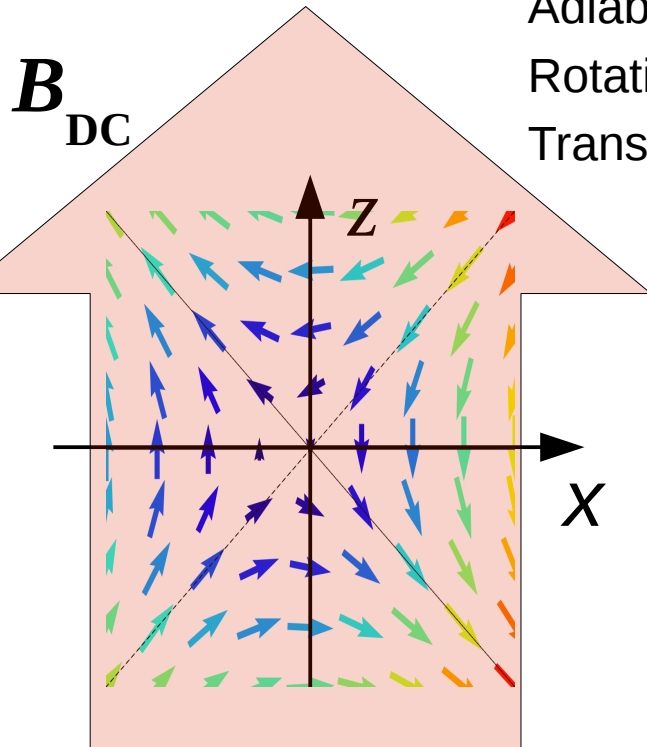
$$H \approx \frac{\mathbf{P}^2}{2m} + (m_F g_F B_{DC} - \hbar \omega) \hat{F}_z + \frac{m_F g_F B_{AC}}{2} \hat{F}_x$$

$$H \approx \frac{\mathbf{P}^2}{2m} + \sqrt{((m_F g_F B_{DC} - \hbar \omega)^2 + \left(\frac{m_F g_F B_{AC}}{2}\right)^2)} \hat{F}_z$$

Adiabatic approximation: Neglect the gauge field  $\mathbf{A}$ .

Rotating Wave Approximation (RWA): neglect the counter rotating term

Transverse field only: neglect misaligned fields.

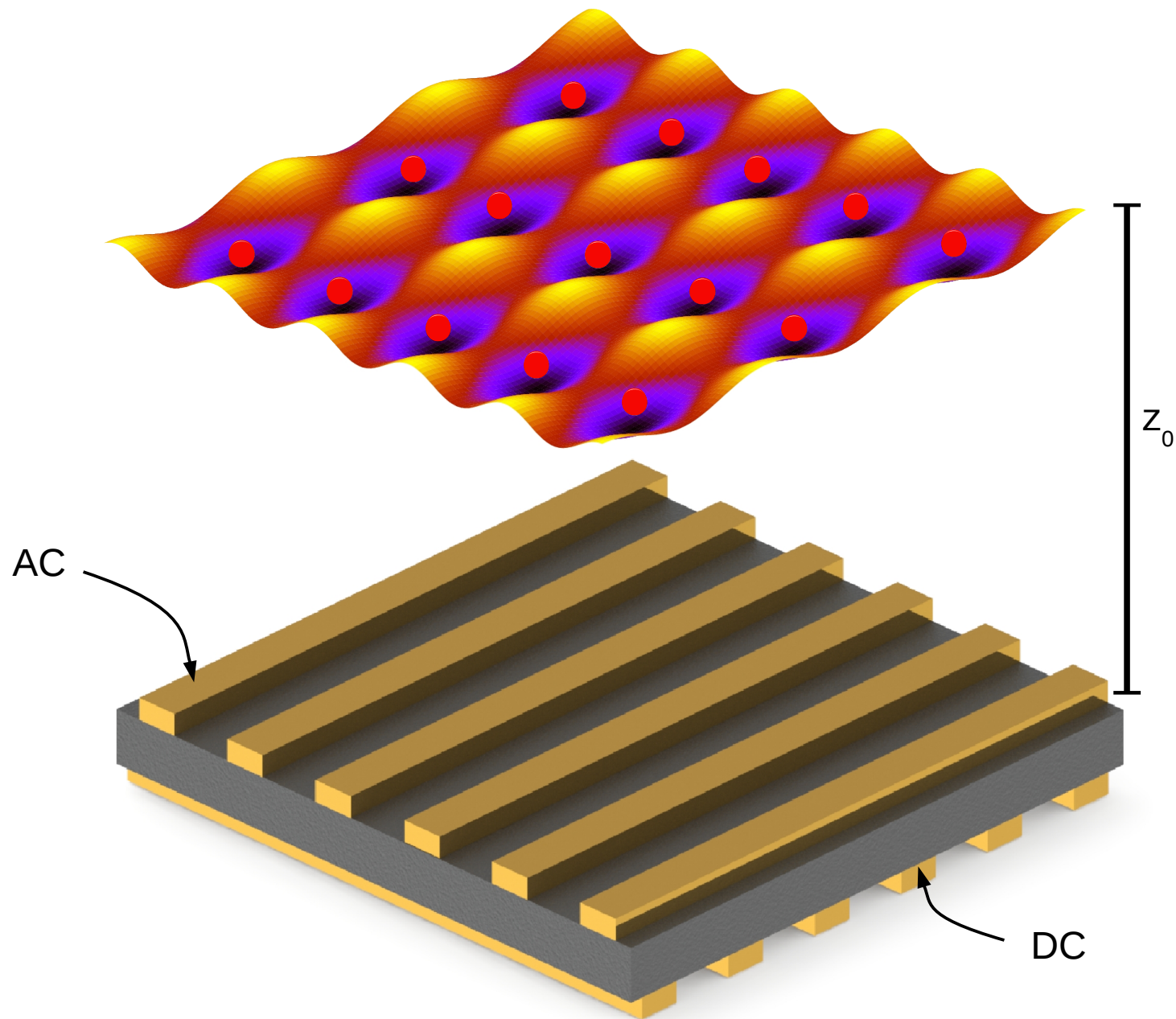


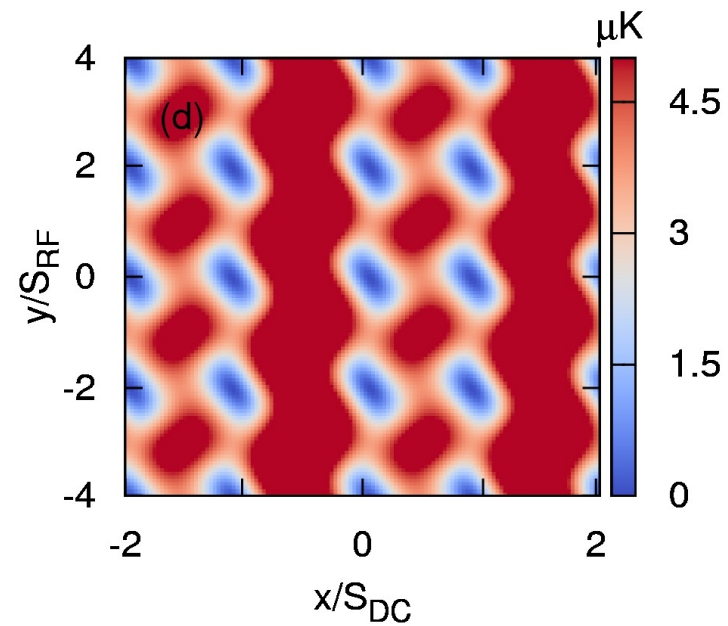
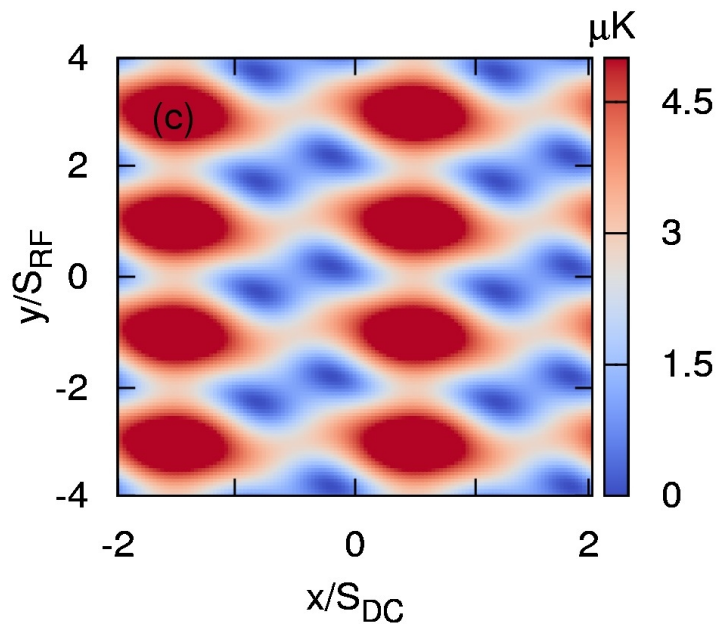
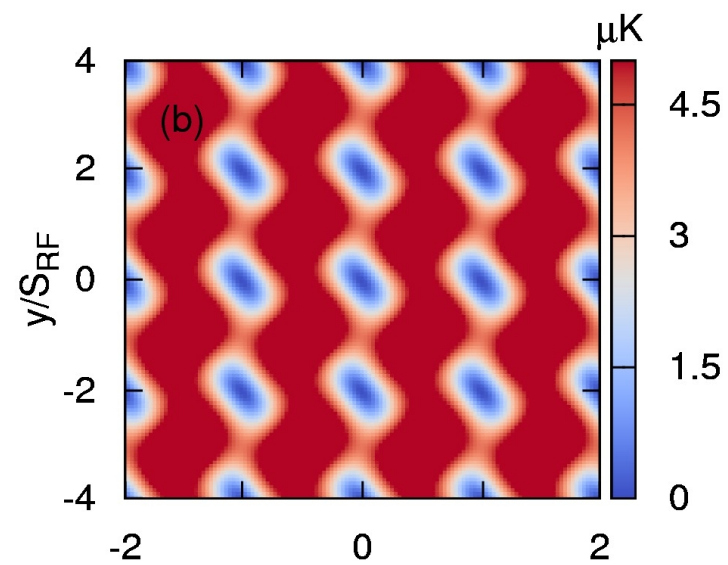
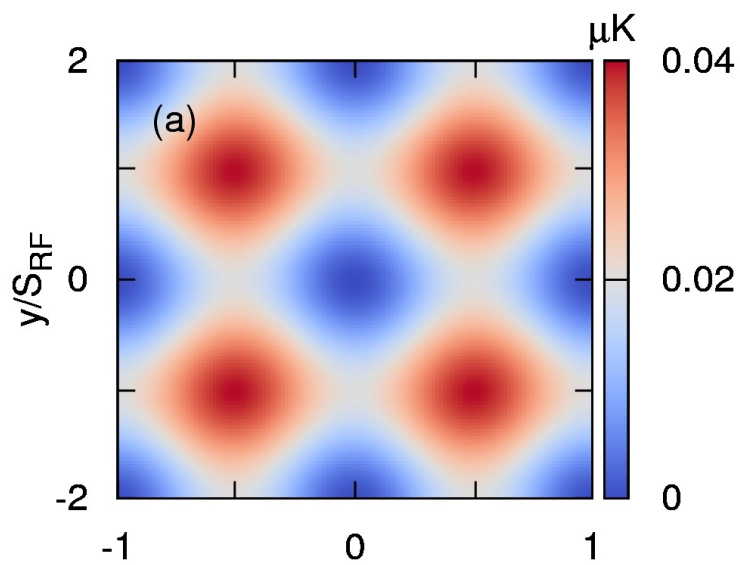
$$H \approx \frac{\mathbf{P}^2}{2m} + (m_F g_F B_{DC} - \hbar \omega) \hat{F}_z + \frac{m_F g_F B_{AC}}{2} \hat{F}_x$$

$$H \approx \frac{\mathbf{P}^2}{2m} + \sqrt{((m_F g_F B_{DC} - \hbar \omega)^2 + \left(\frac{m_F g_F B_{AC}}{2}\right)^2)} \hat{F}_z$$

$$V_{adb}(r) = m_F \sqrt{(\text{Detuning})^2 + \left(\frac{1}{2} \text{Rabi Frequency}\right)^2}$$



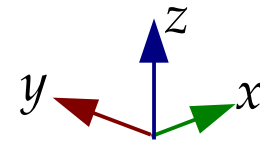
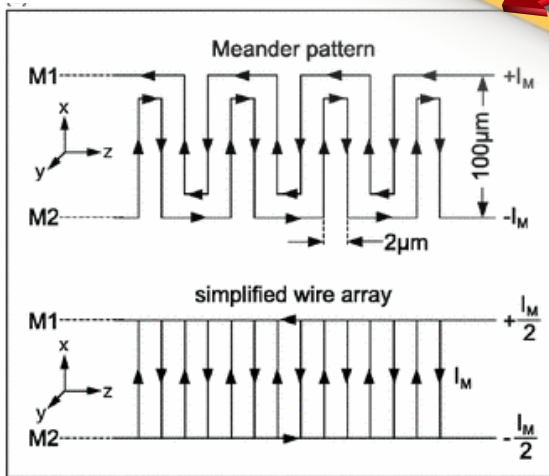
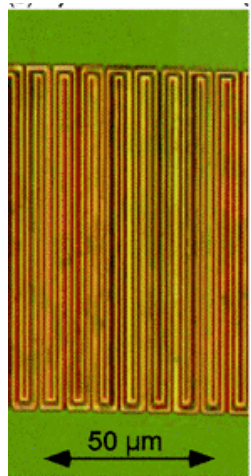
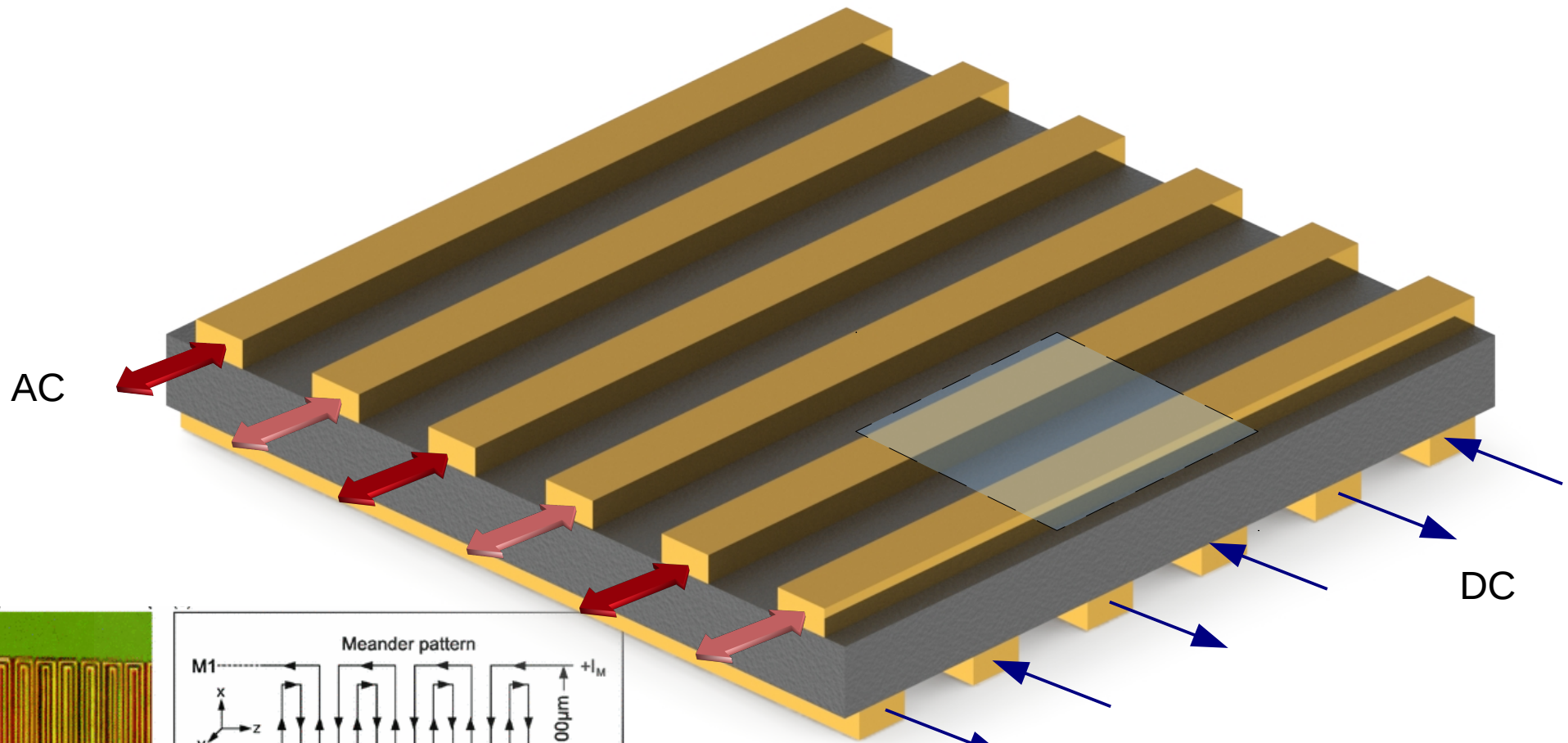




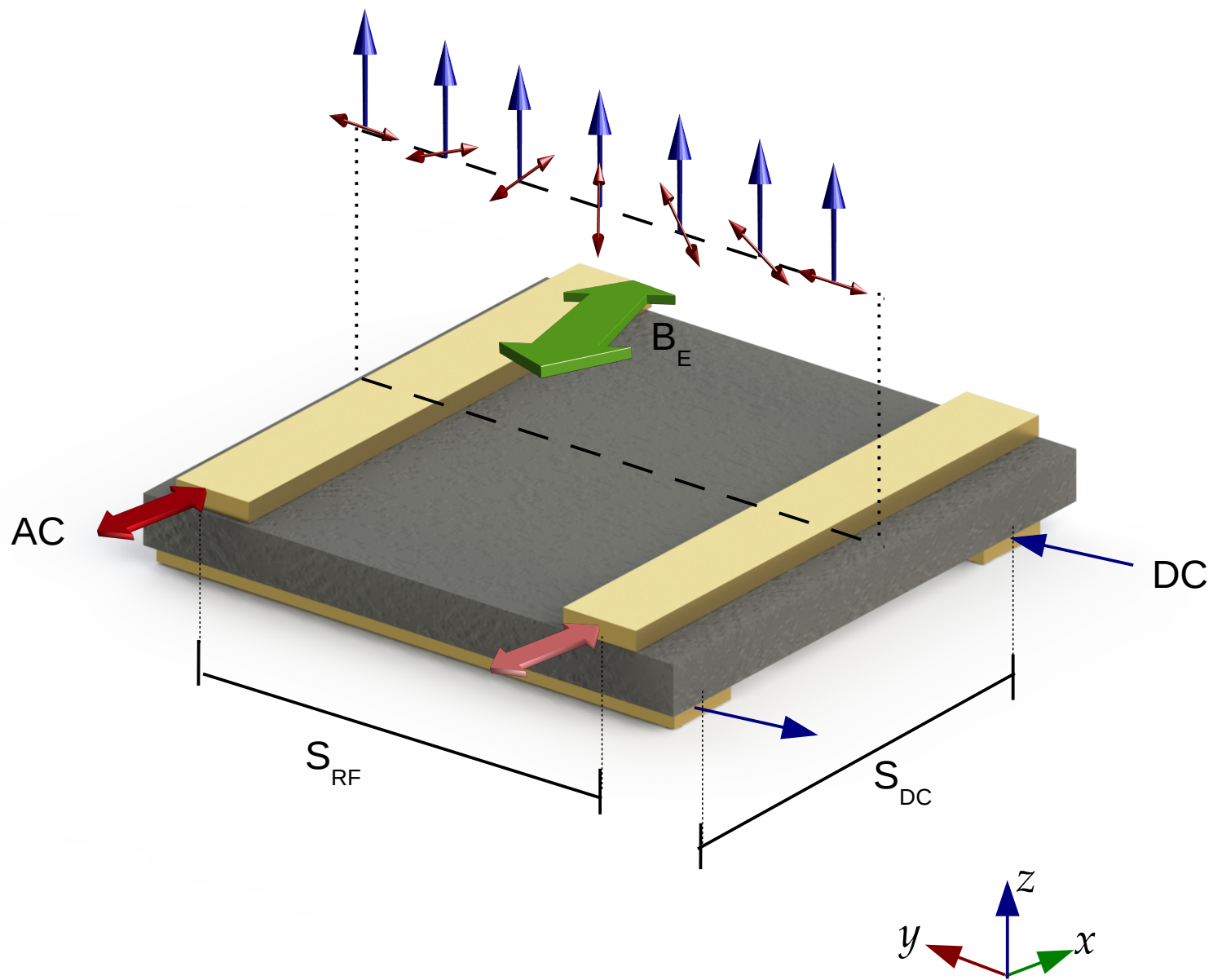
# Dressed 2D Lattice

Atom-chip design of a dressed 2D periodic potential:

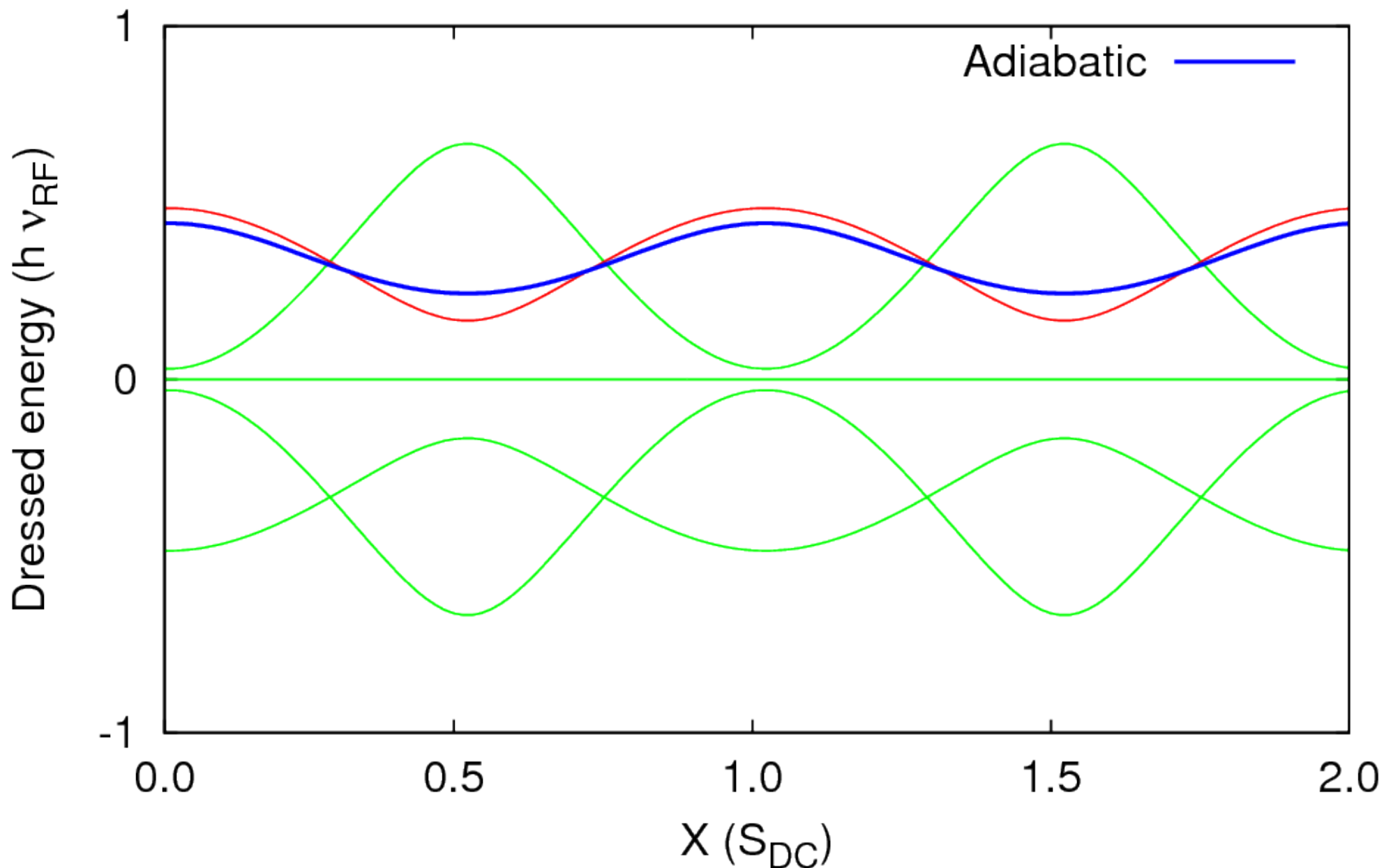
- Highly controllable configuration: complex periodic potential from using a simple conductor layout.
- Large trapping frequencies in close proximity to chip surface.
- Simultaneous trapping of two hyperfine states: Microwave coupling can be used for applications as in Optical lattices.



Gunther et al. PRA 2005

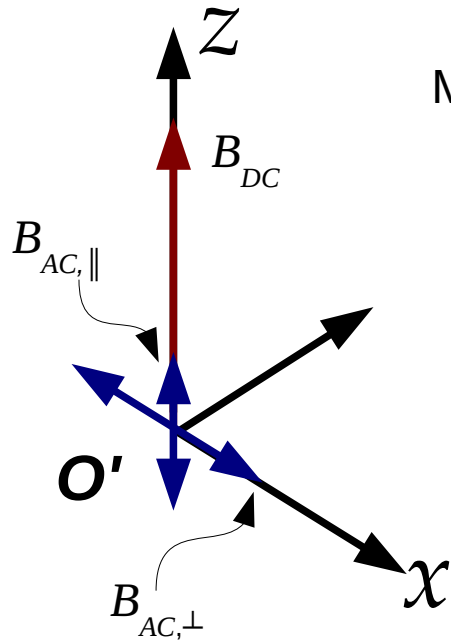


# Resonant driving, Weak RF



Pegg and Series, Proc. R. Soc. Lond.A 332, 281 (1973).

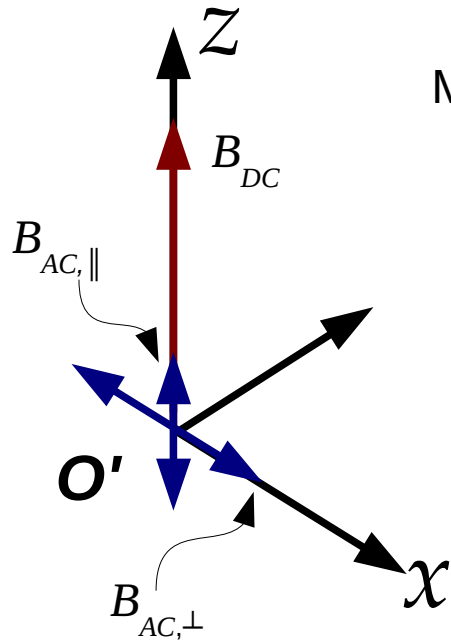
Misalignment of the dressing field:



Pegg and Series, Proc. R. Soc. Lond.A 332, 281 (1973).

Misalignment of the dressing field:

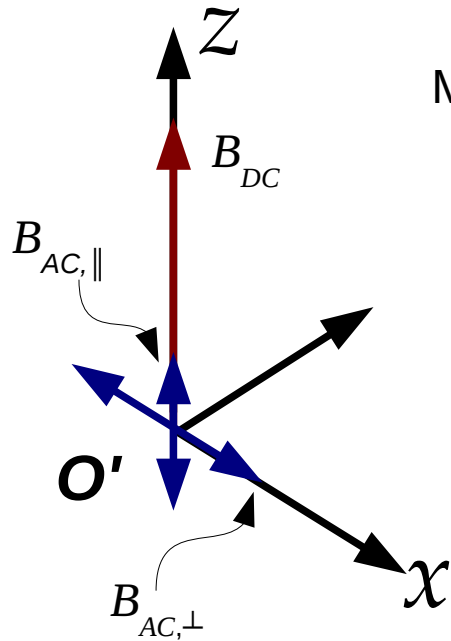
$$U(t) = \exp(-i\omega t \hat{F}_z)$$





Pegg and Series, Proc. R. Soc. Lond.A 332, 281 (1973).

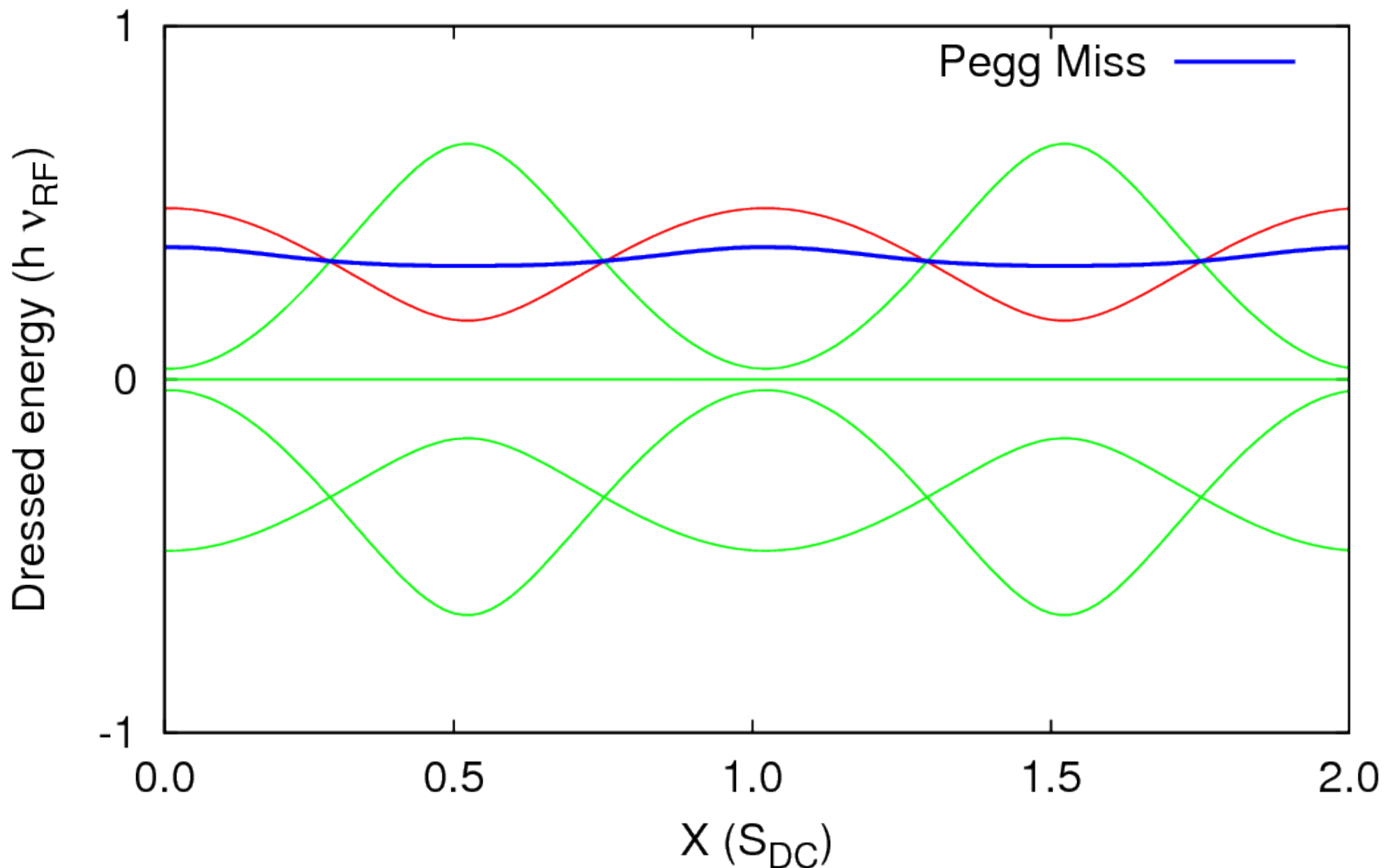
Misalignment of the dressing field:



$$U(t) = \exp \left( -i \left( (q_0 + 1) \omega t + \frac{\mu_B g_F B_{AC}^z}{\hbar \omega} \sin \omega t \right) \hat{F}_z \right)$$

$$q_0 \hbar \omega + \mu_B g_F B_{DC}^z \approx 0$$

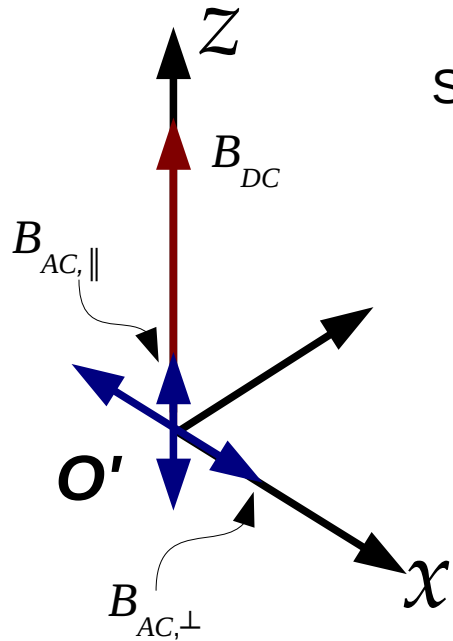
# Resonant driving, Weak RF



Pegg and Series, Proc. R. Soc. Lond.A 332, 281 (1973).

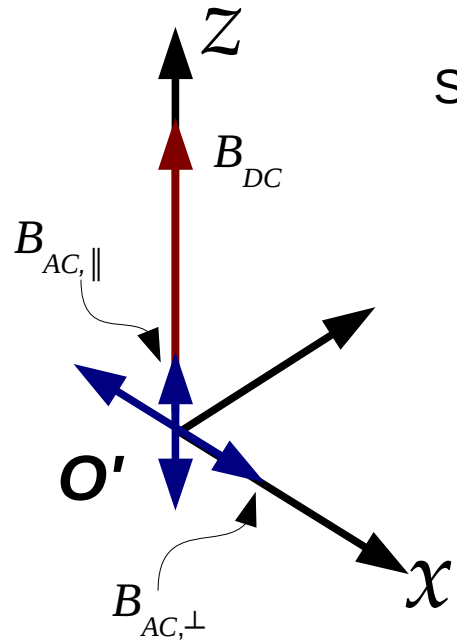
Strong dressing field:

$$U(t) = \exp(-i\omega t \hat{F}_z)$$



Pegg and Series, Proc. R. Soc. Lond.A 332, 281 (1973).

Strong dressing field:



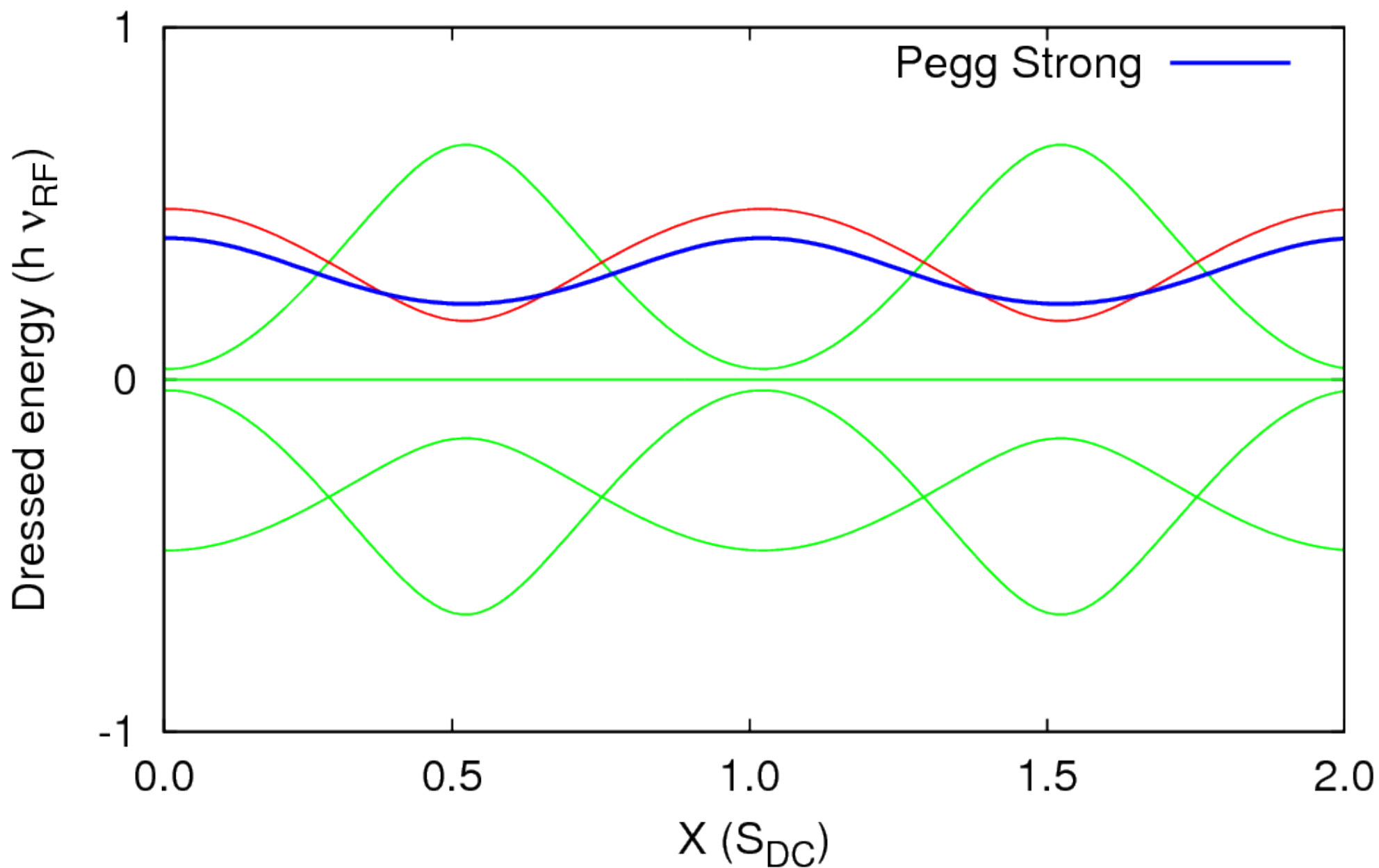
$$U(t) = U_1 R_Y(\theta) U_2$$

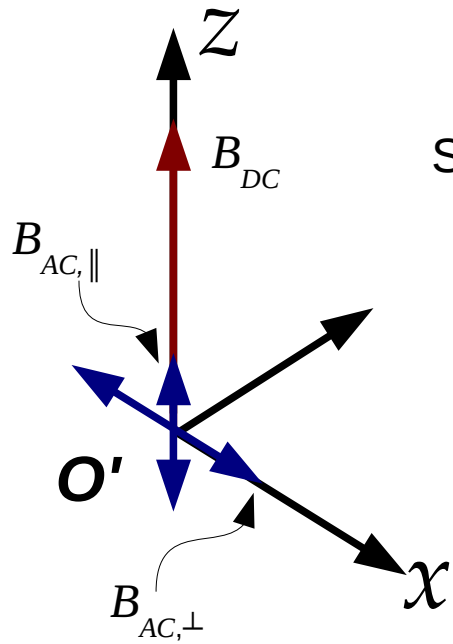
$$U_1(t) = \exp(i\omega t \hat{F}_z)$$

$$U_2(t) = \exp\left(-i\left(2(q_0 + 1)\omega t + \frac{\mu_B g_F B_{AC}^x}{4\hbar\omega} \sin\theta \sin 2\omega t\right)\hat{F}_z\right)$$

$$q_0 \hbar\omega + \mu_B g_F B_{DC}^z \approx 0$$

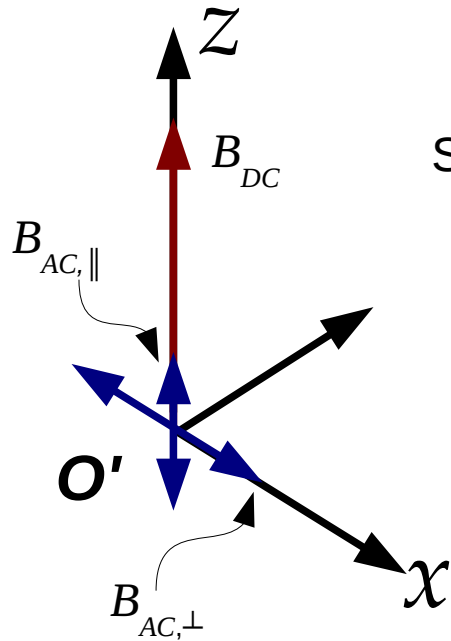
# Resonant driving, Weak RF





Strong dressing + misalignment

$$U(t) = \exp(-i\omega t \hat{F}_z)$$



Strong dressing + misalignment

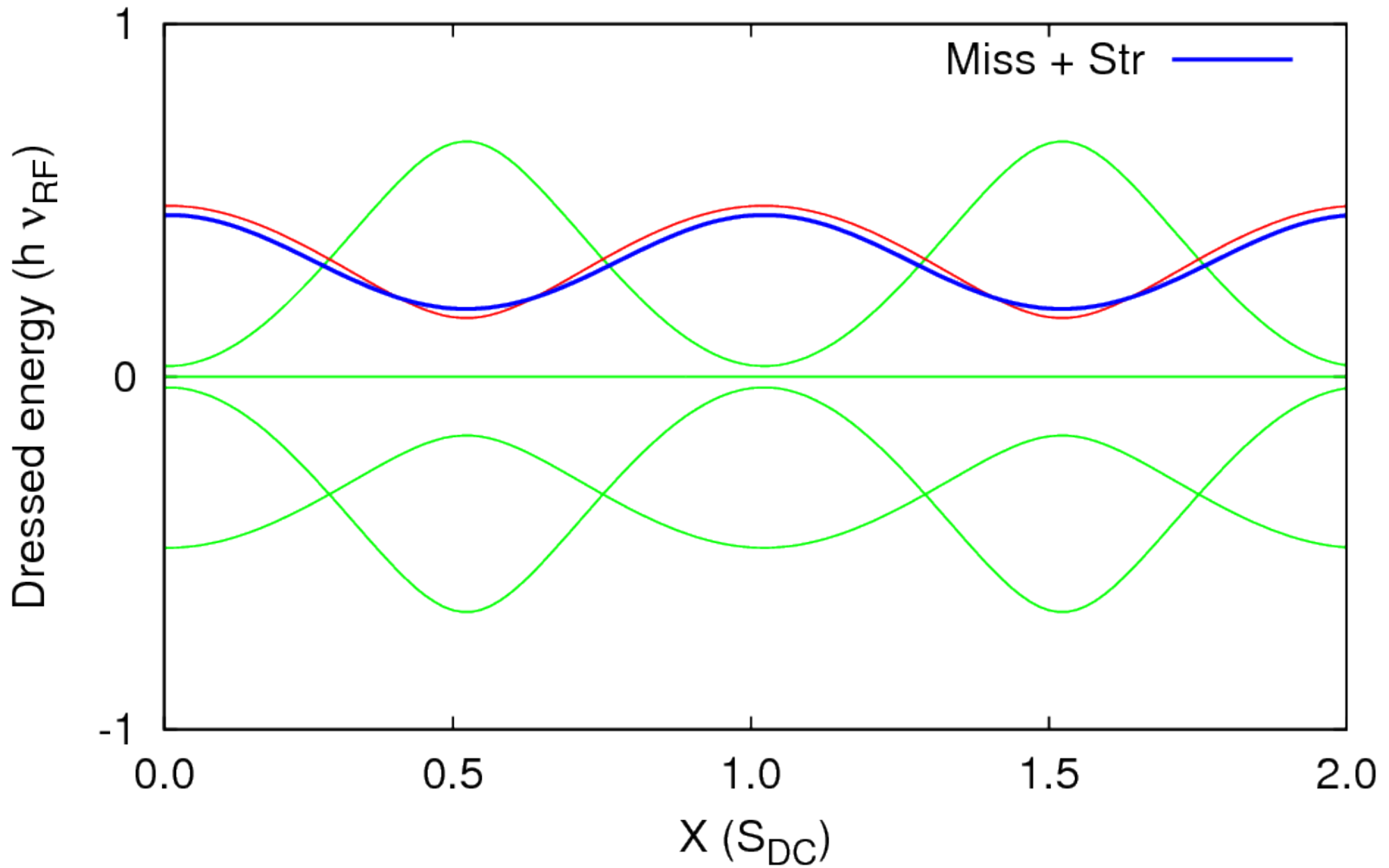
$$U(t) = U_1 R_Y(\theta) U_2$$

$$U_1(t) = \exp(i \omega t \hat{F}_z)$$

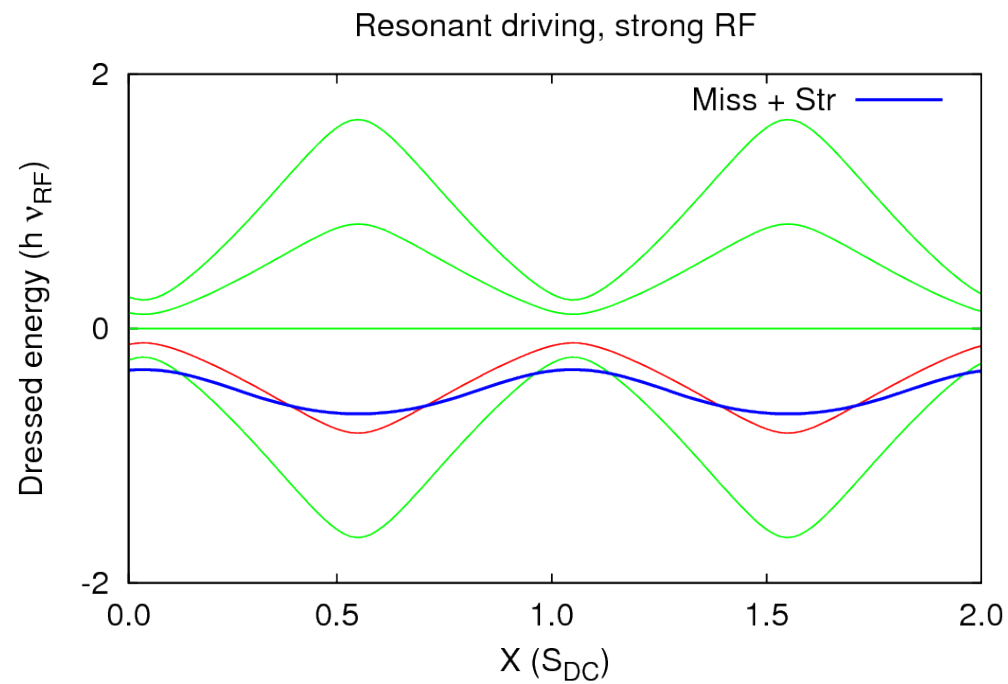
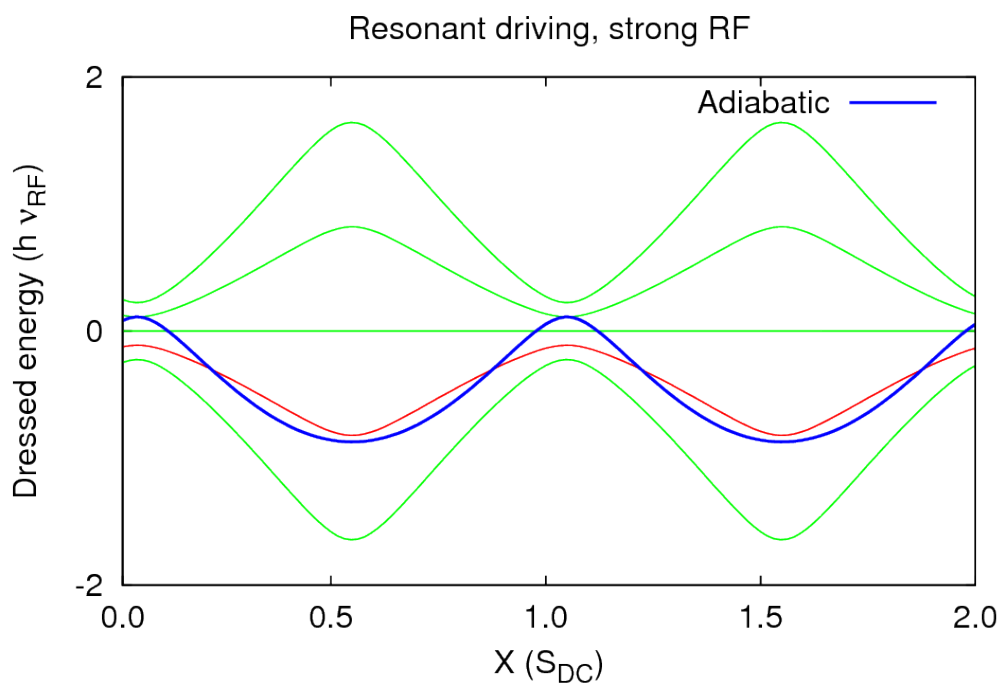
$$U_2(t) = \exp \left( -i \left( 2(q_0 + 1) \omega t + \frac{\mu_B g_F B_{AC}^x}{4 \hbar \omega} \sin \theta \sin 2 \omega t + \frac{\mu_B g_F B_{AC}^z}{\hbar \omega} \cos \theta \sin \omega t \right) \hat{F}_z \right)$$

$$q_0 \hbar \omega + \mu_B g_F B_{DC}^z \approx ?$$

# Resonant driving, Weak RF







Q: Is there a general procedure to find a frame of reference where the time-dependent effects can be neglected?

$$U(t) = ?$$

A: Possibly.

Quantum optics: Integrability of the two-level Rabi problem  
D. Braak, PRL 107, 100401 (2011)

Floquet-Magnus expansion: Effective Hamiltonian as a power series in  $1/\omega$

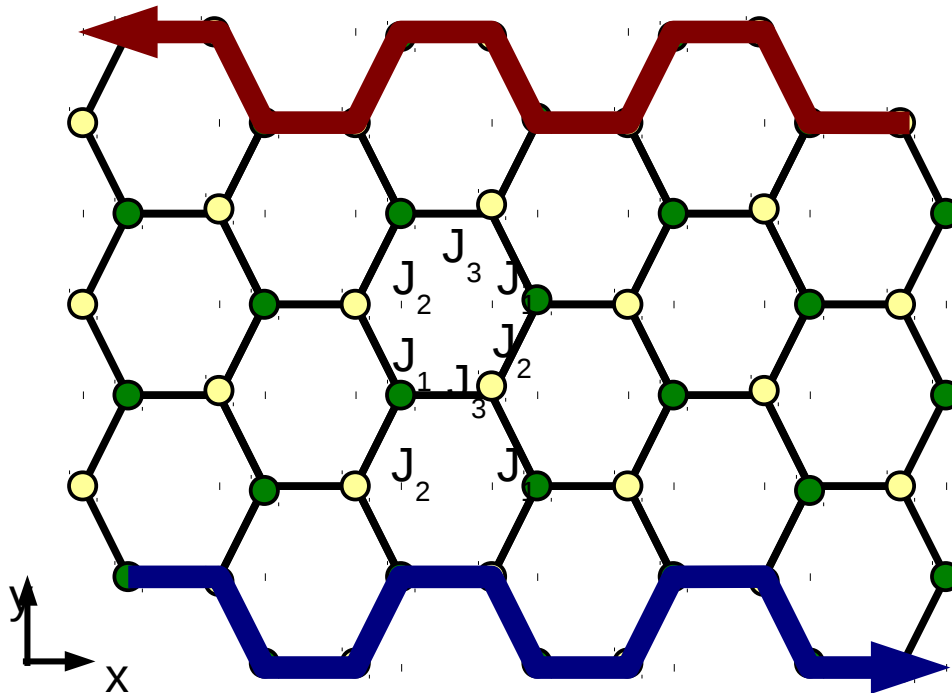
Unitary flow in Floquet space: Application of flow equations to periodic Hamiltonian in the interaction picture.  
PRL 111, 175301 (2013).

## IV. 2D Band engineering with periodic driving

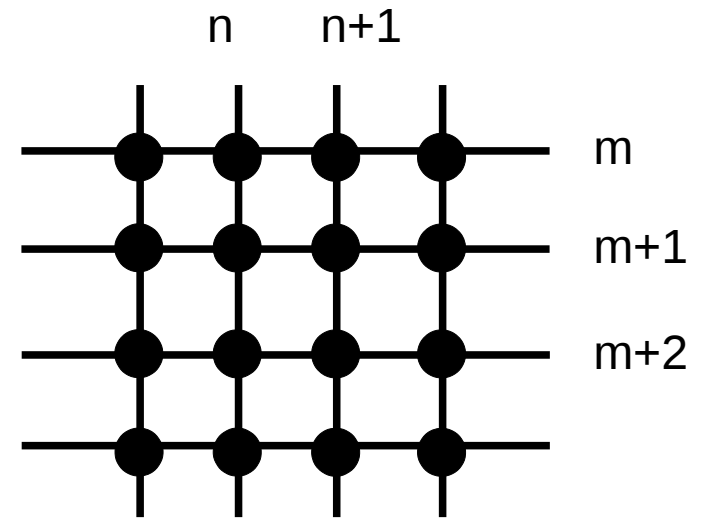
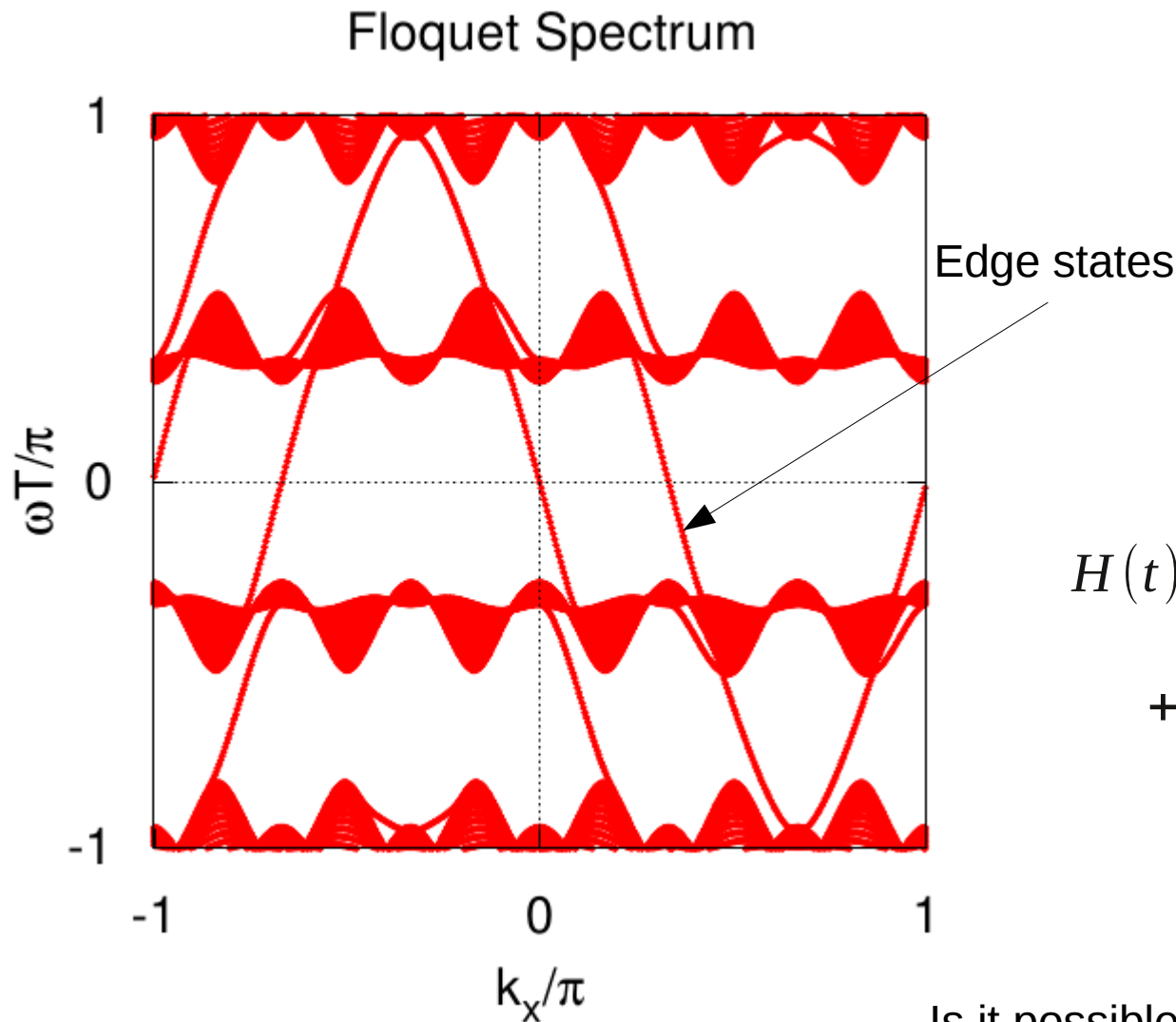
## Floquet Topological Insulator:

Shining light on conventional insulators produces a system whose effective energy bands have a non-trivial topology. (Linder, Nat. Phys. **7**, 490 (2011))

The effective Hamiltonian associated with fast periodic driving of lattice models contains terms with long-range hopping, and can resemble the Haldane Hamiltonian (Demler, PRB (2011)).



$$H_{eff} = H_0 + \frac{1}{\hbar \omega} [V_{-1}, V_1] + \dots$$



$$H(t) = \sum_{n,m} J_x(t) a_{n,m}^\dagger a_{n+1,m} + h.c.$$

$$+ J_y(t) \exp(i\alpha n) a_{n,m}^\dagger a_{n,m+1} + h.c.$$

Is it possible to count the number of edge states from the structure of the energy bands? (bulk-edge correspondence)

There have been various attempts to solve this problem:

Homotopy invariant: Demler, PRB (2011)

Winding number: Levin, PRX (2013)

Topological Charges: Jiang, PRL (2011)

But, there are situations where none of these quantities predict correctly the number of edge modes.

One particularly challenging example is the Hofstadter Model where one of the tunneling constants varies periodically (Zhao, PRL, PRA 2014).

What about the entanglement spectrum for FTI?

# V. Conclusion

How to extend the theory of quantum pumping to regimes of fast driving and interacting?

Does strong dressing brings new types of dressed potential landscapes?

Can the Floquet spectrum in the bulk determine the number of edge modes in lattice models?

Several applications will be benefit of developing techniques for finding the Floquet operator of time-periodic systems

The Floquet formalism offers a common language for a range dissimilar problems. Thus, developments in one area can be immediately translated/adapted to other situations.

# Collaborations



University of Sussex



Prof. Barry Garraway  
Miss Kathryn Burrows



Dr. Aidan Arnold



Ruhr Universität Bochum  
Dr. Arcesio Castaneda

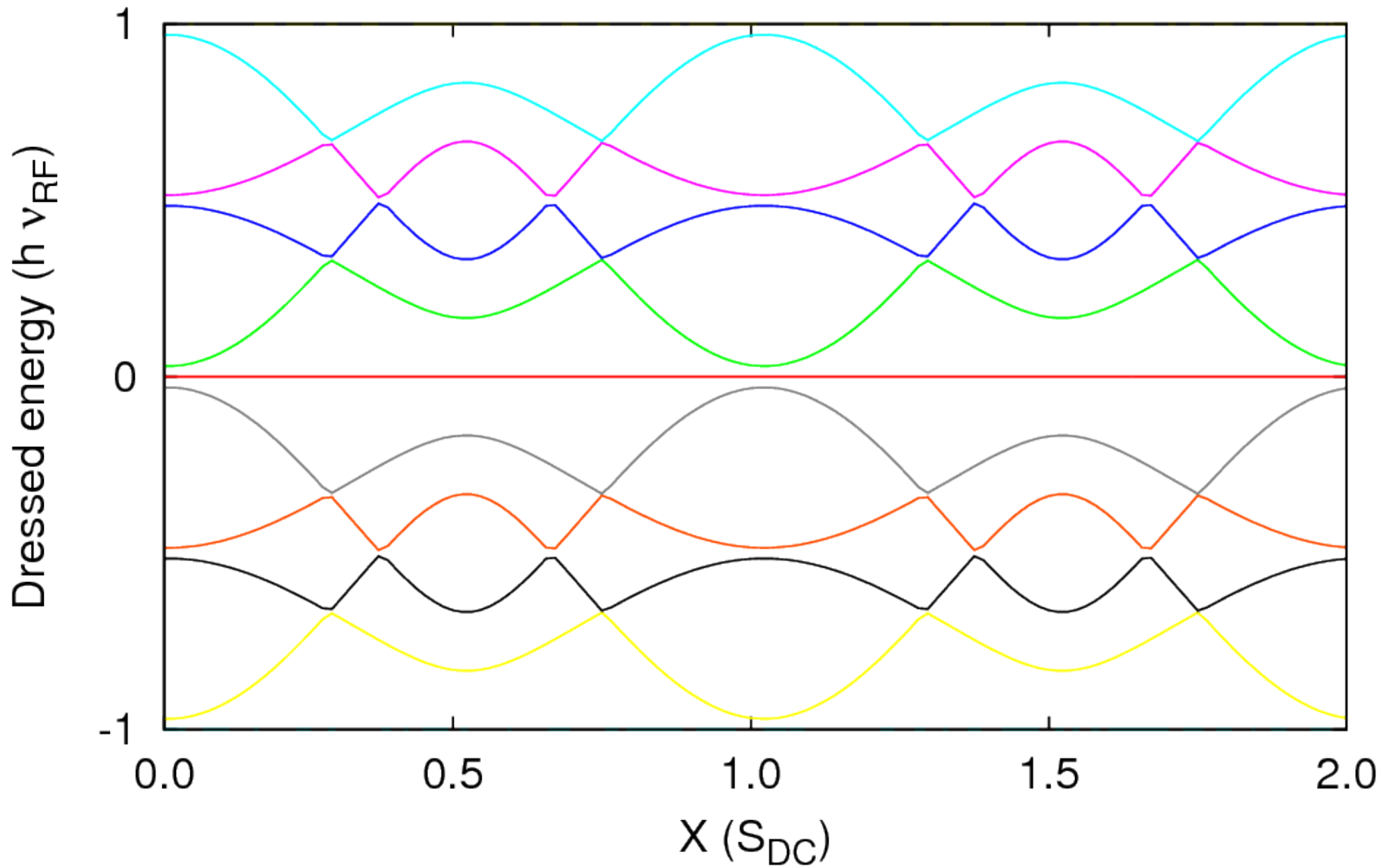


Prof. Thomas Dittrich

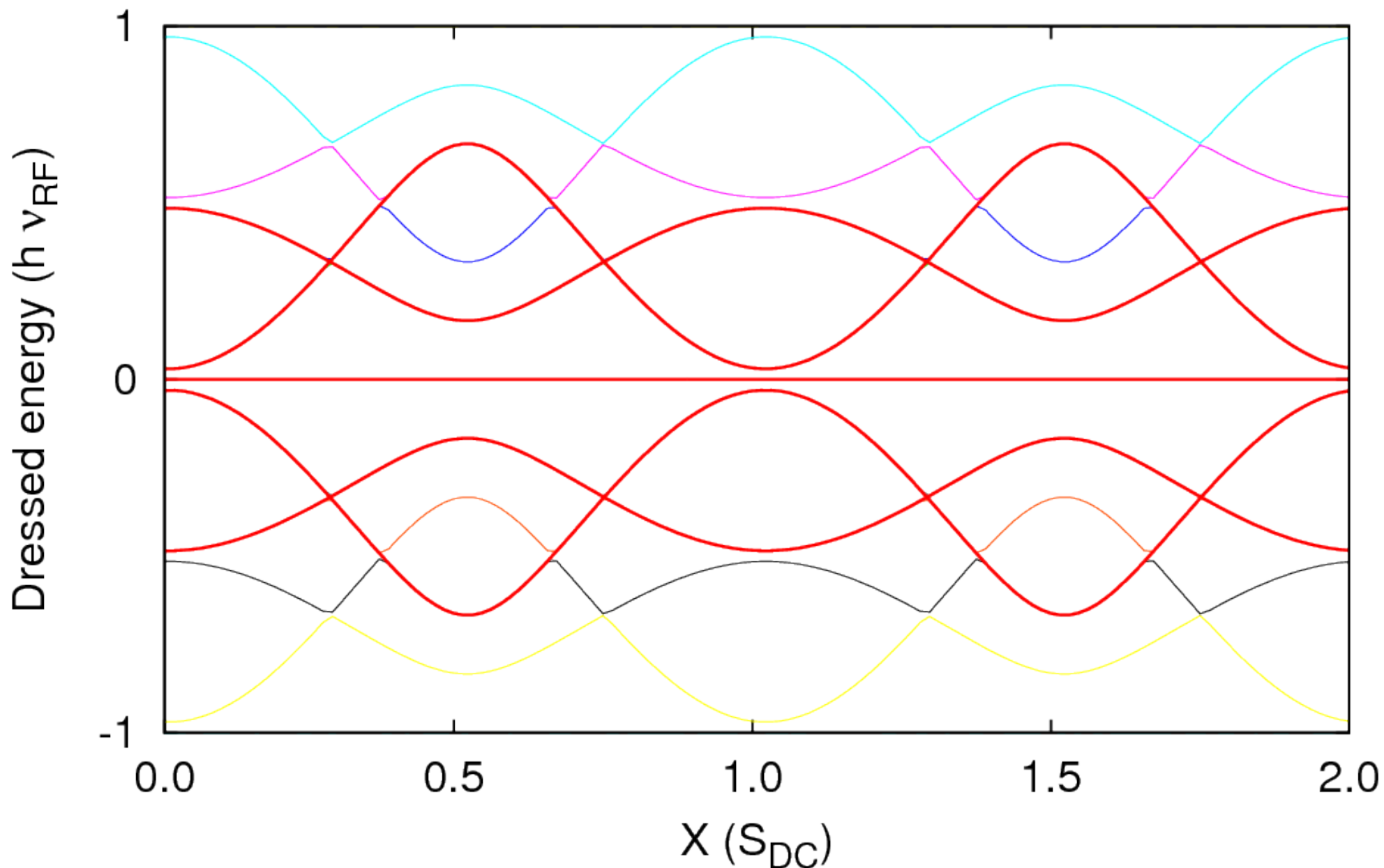




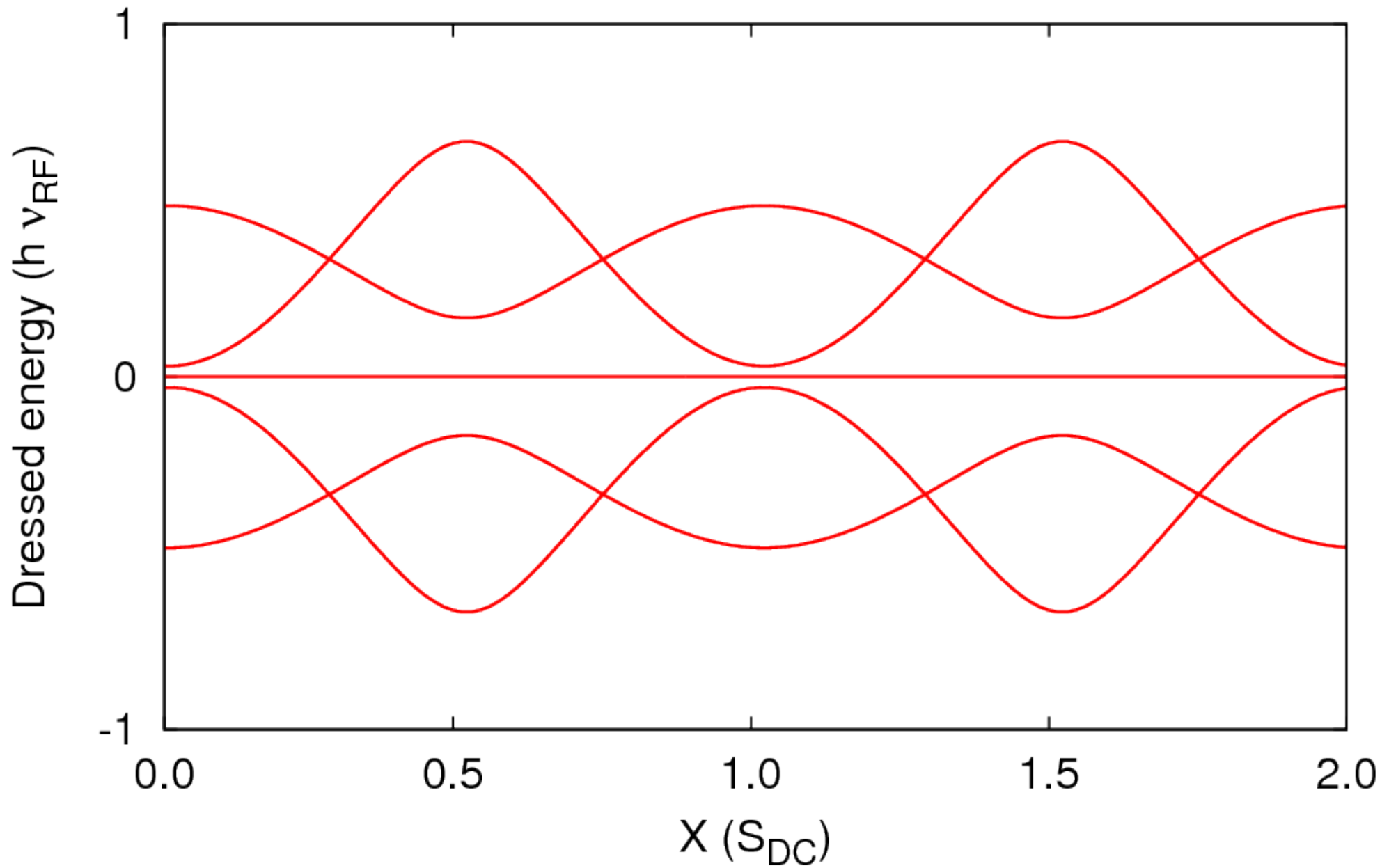
# Resonant driving, Weak RF



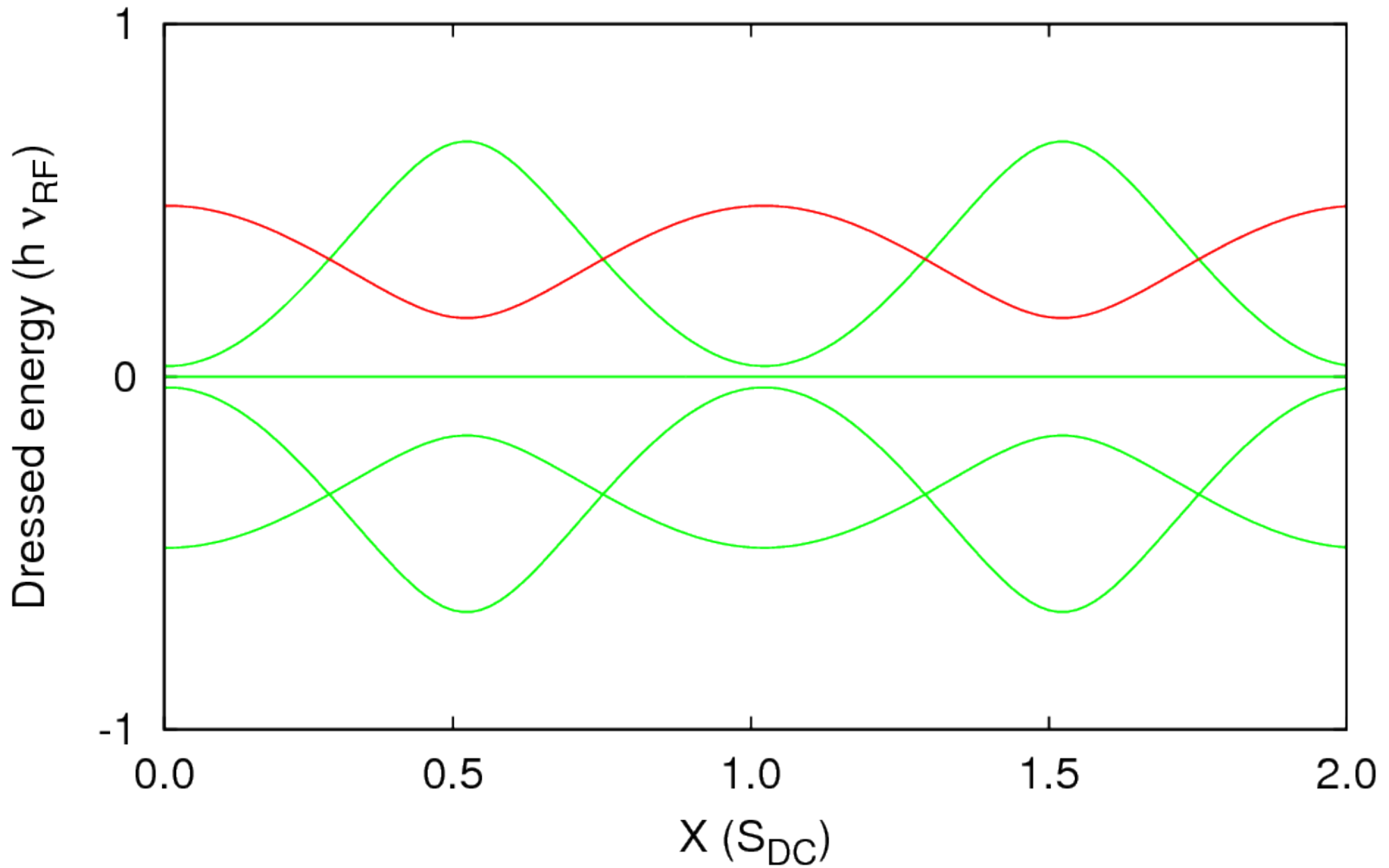
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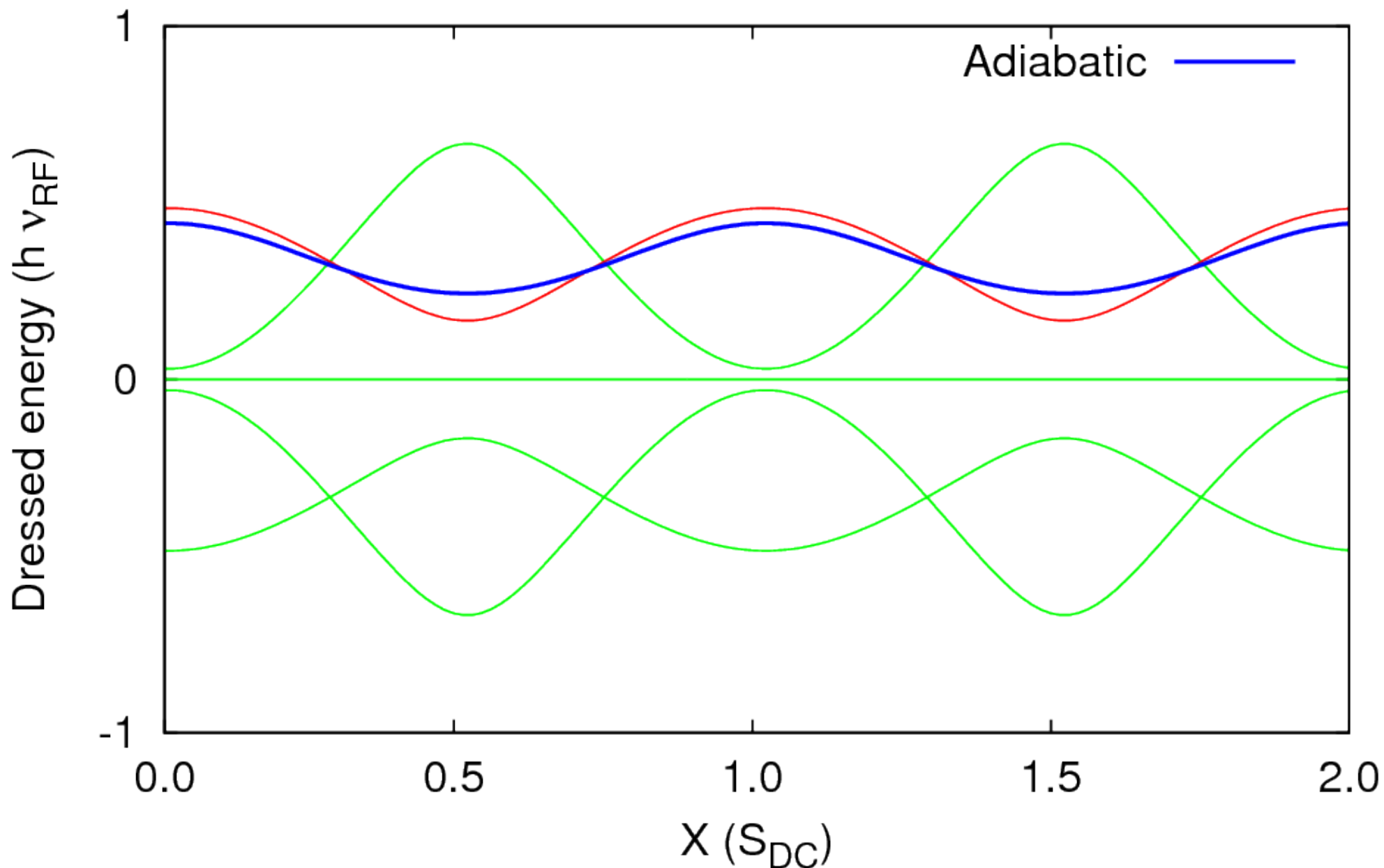
## Resonant driving, Weak RF



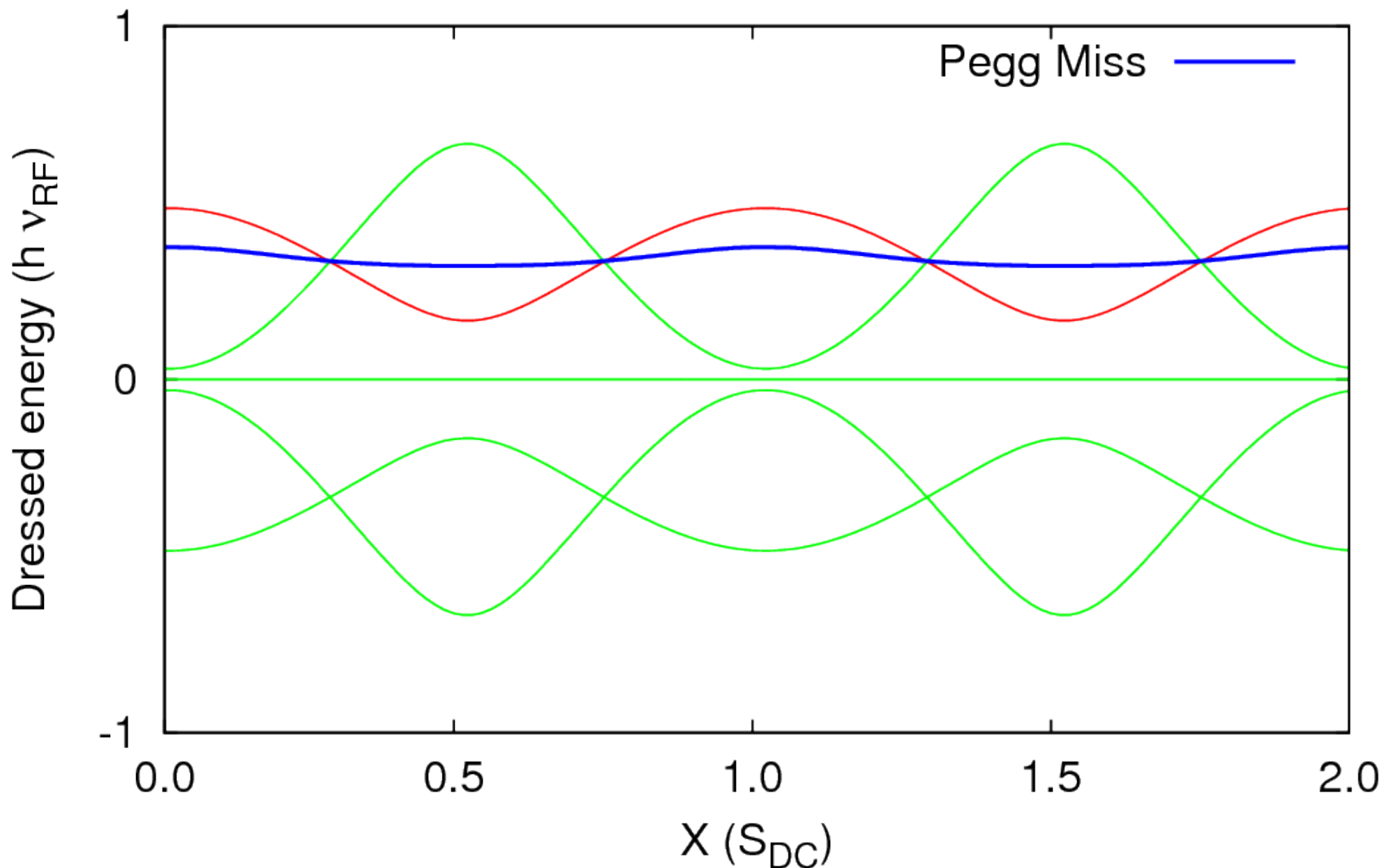
## Resonant driving, Weak RF



# Resonant driving, Weak RF



# Resonant driving, Weak RF



Pegg and Series 1960, for NMR: strong and misalignment

Combination:

Open question. This is not only to evaluate the potential landscape  
But also useful to estimate non-adiabatic effects. This last talk  
cannot be performed straightforwardly using the numerically exact  
spectrum

Relate to Rabi problem: Bargmann or continued fractions, Floquet-  
Magnus expansion, renormalization

To evaluate the Floquet Hamiltonian

Fast driving  $\rightarrow$  Floquet operator:

Change the nature of the system: Galitski

2D lattice...

Open problem: bulk-edge correspondence.

Hyp: Entanglement spectrum



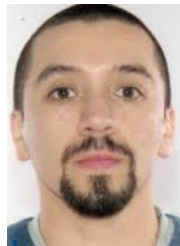
# Collaborations



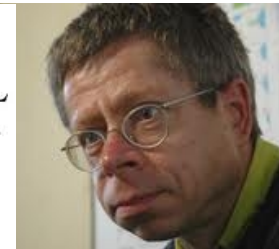
Prof. Barry Garraway  
Miss Kathryn Burrows



Dr. Aidan Arnold



Ruhr Universität Bochum  
(Dr.) Arcesio Castaneda



Prof. Thomas Dittrich



Kathryn Burrows,

University of Sussex, Brighton, UK

Barry Garraway

and

Aidan Arnold

University of Strathclyde, Glasgow, UK

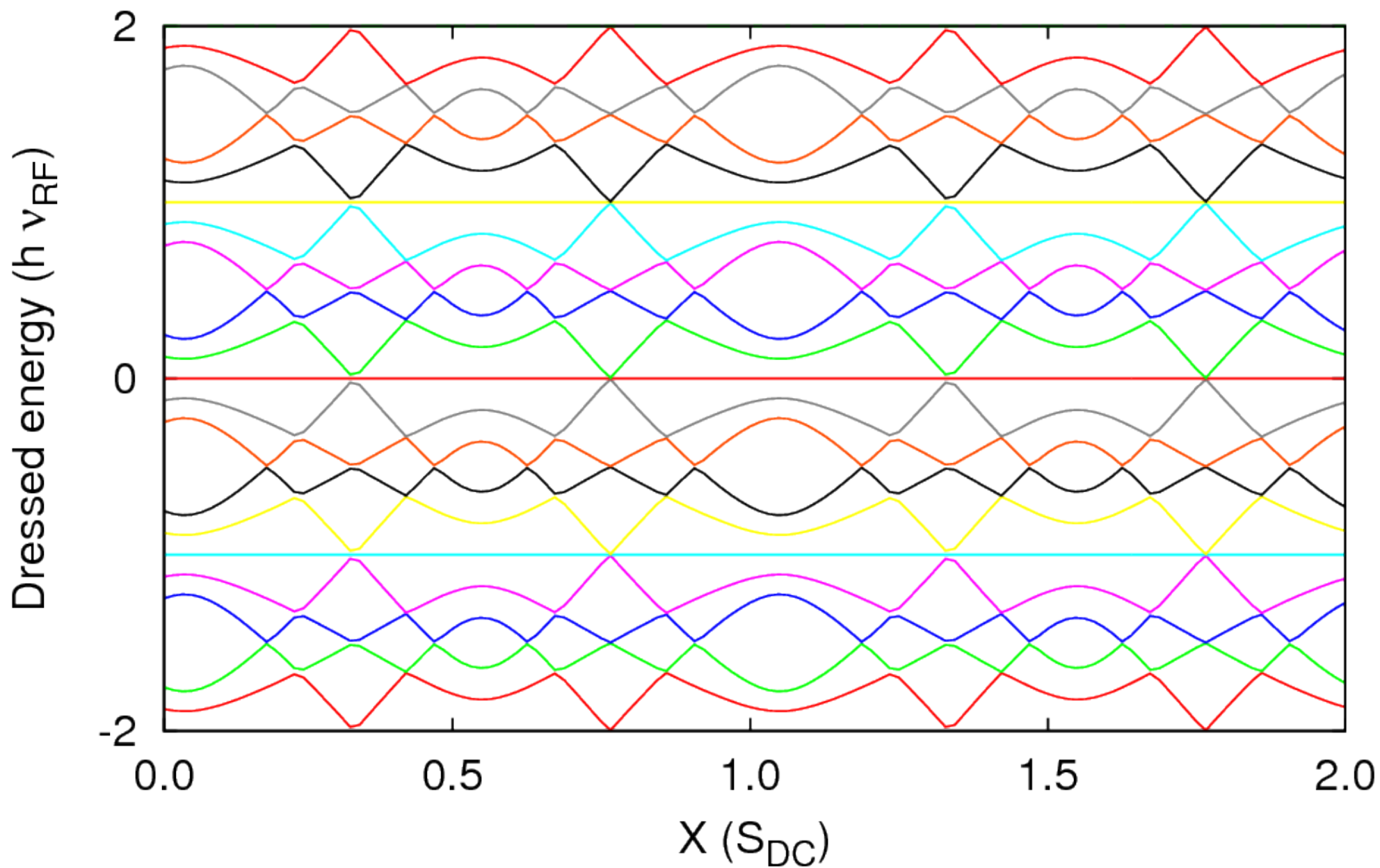
Thomas Dittrich

National University of Colombia, Bogota, Colombia.

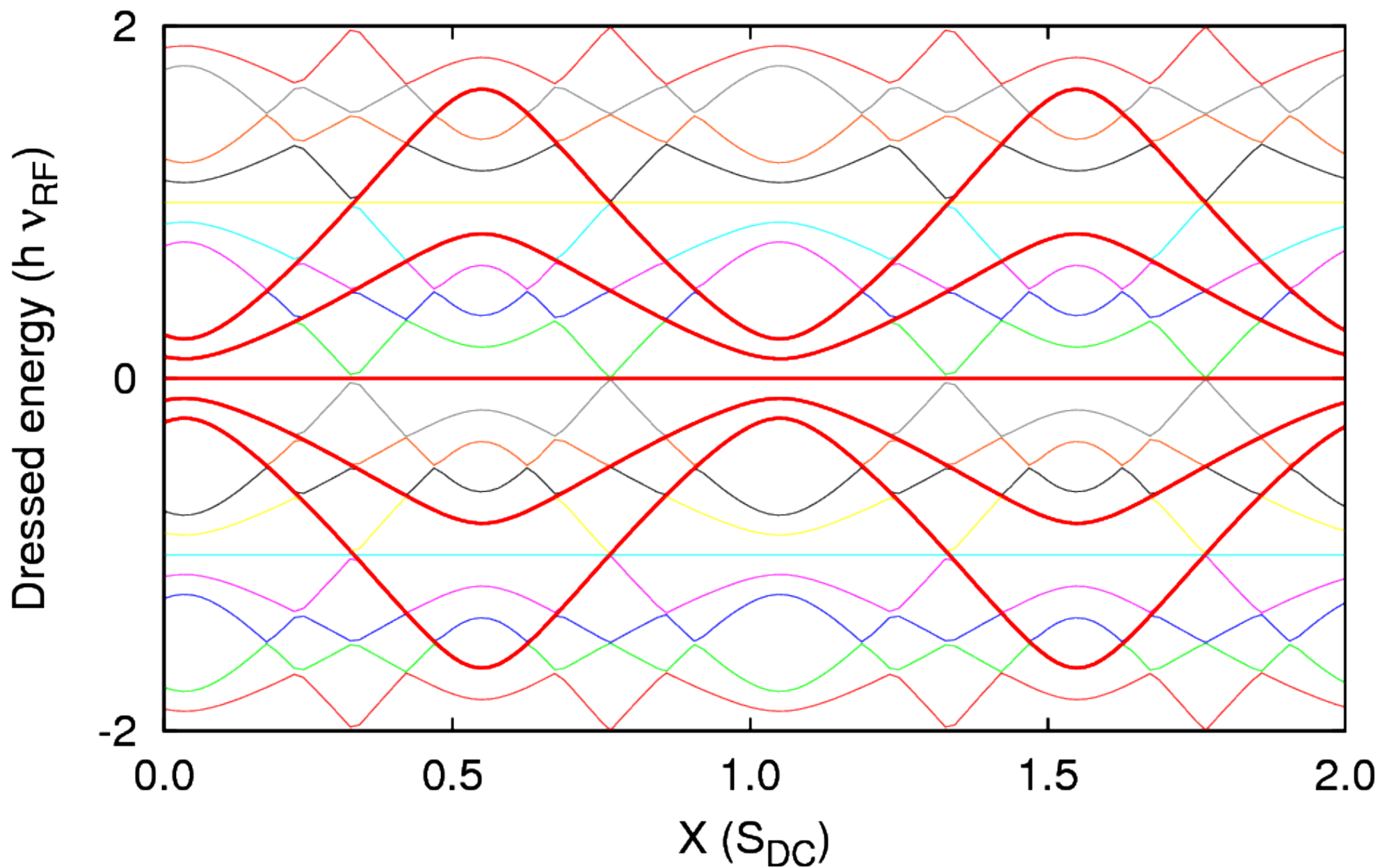
and

Arcesio Castaneda

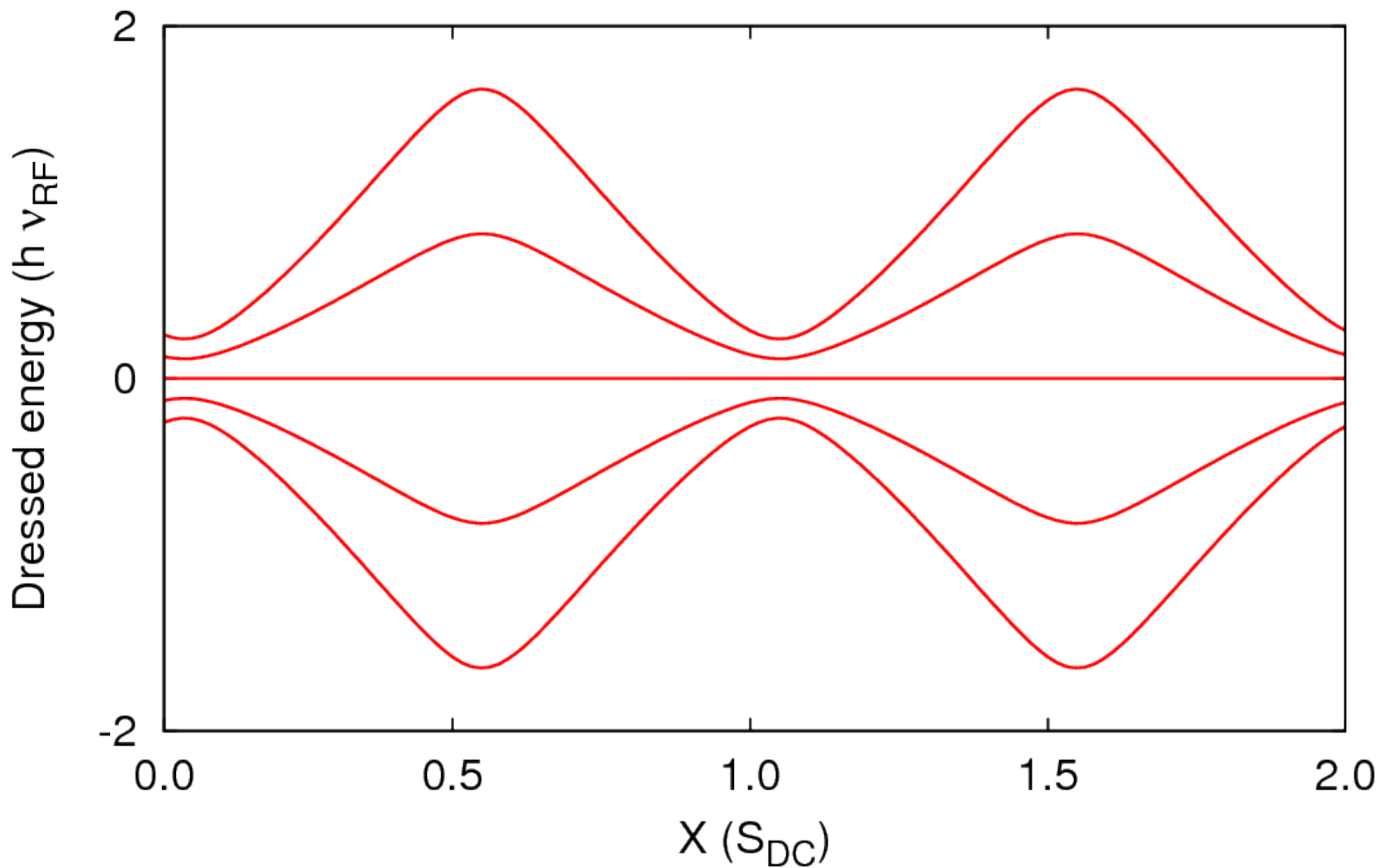
# Resonant driving, strong RF



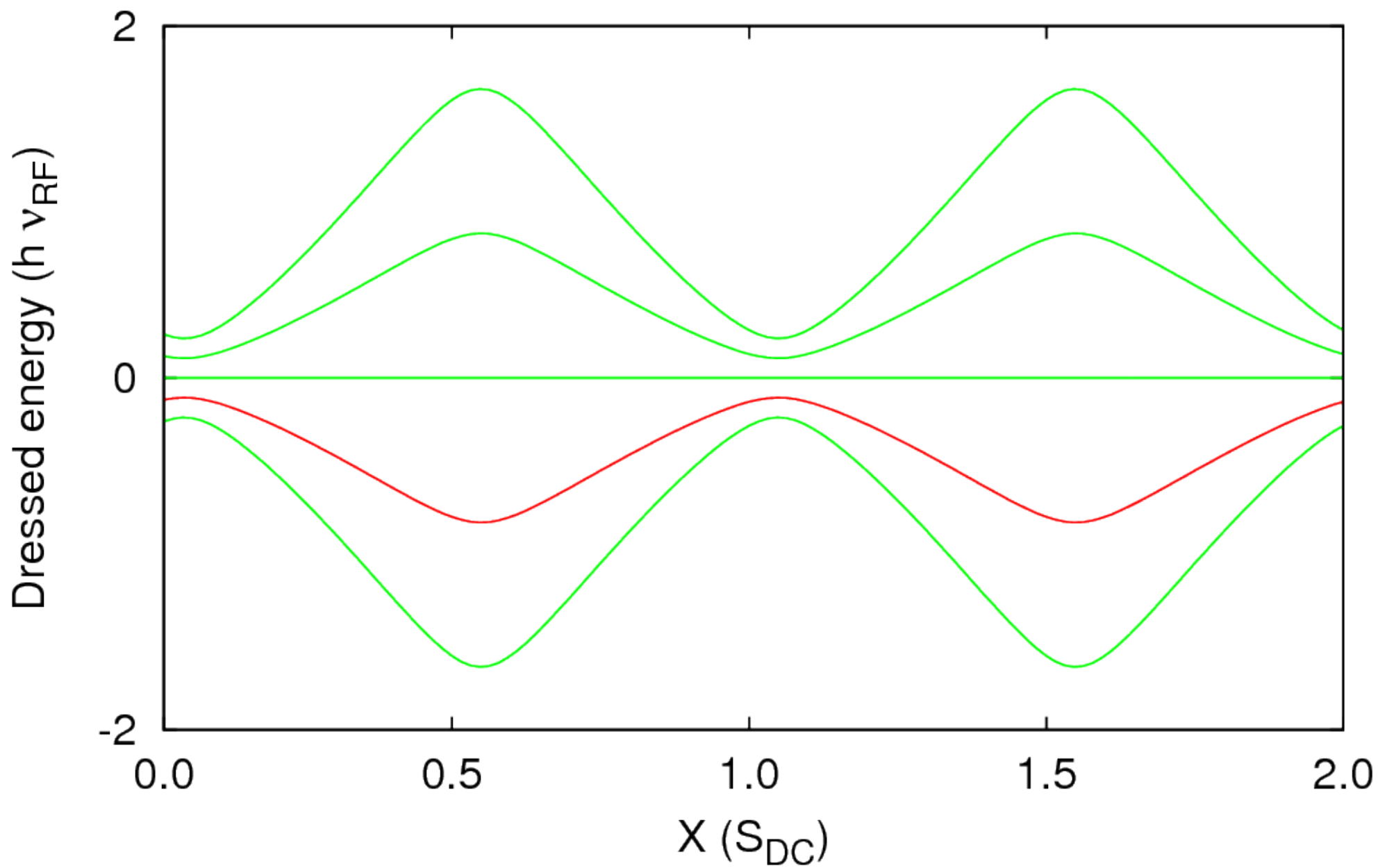
# Resonant driving, strong RF



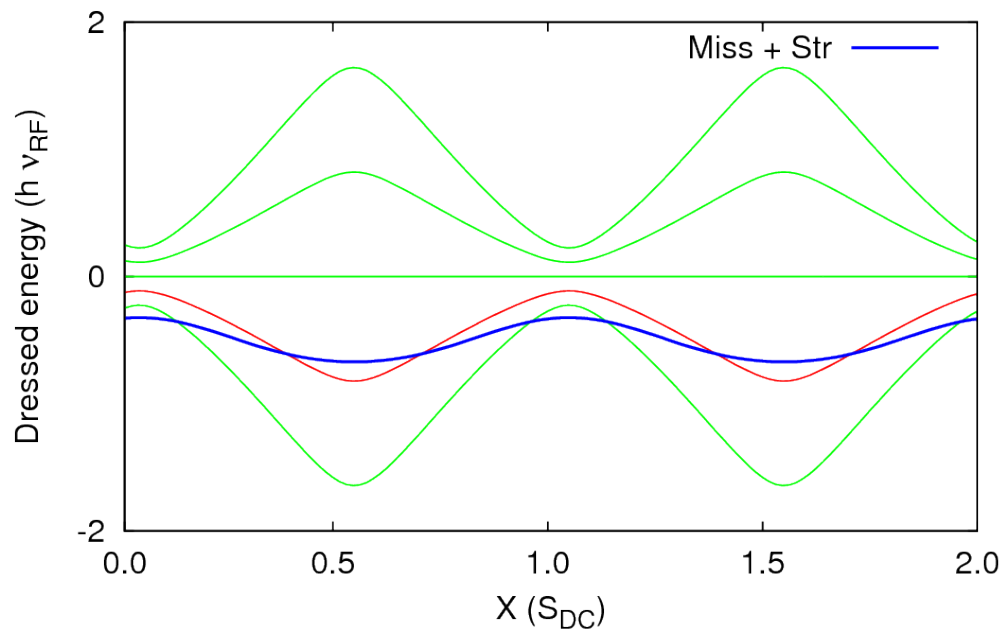
# Resonant driving, strong RF



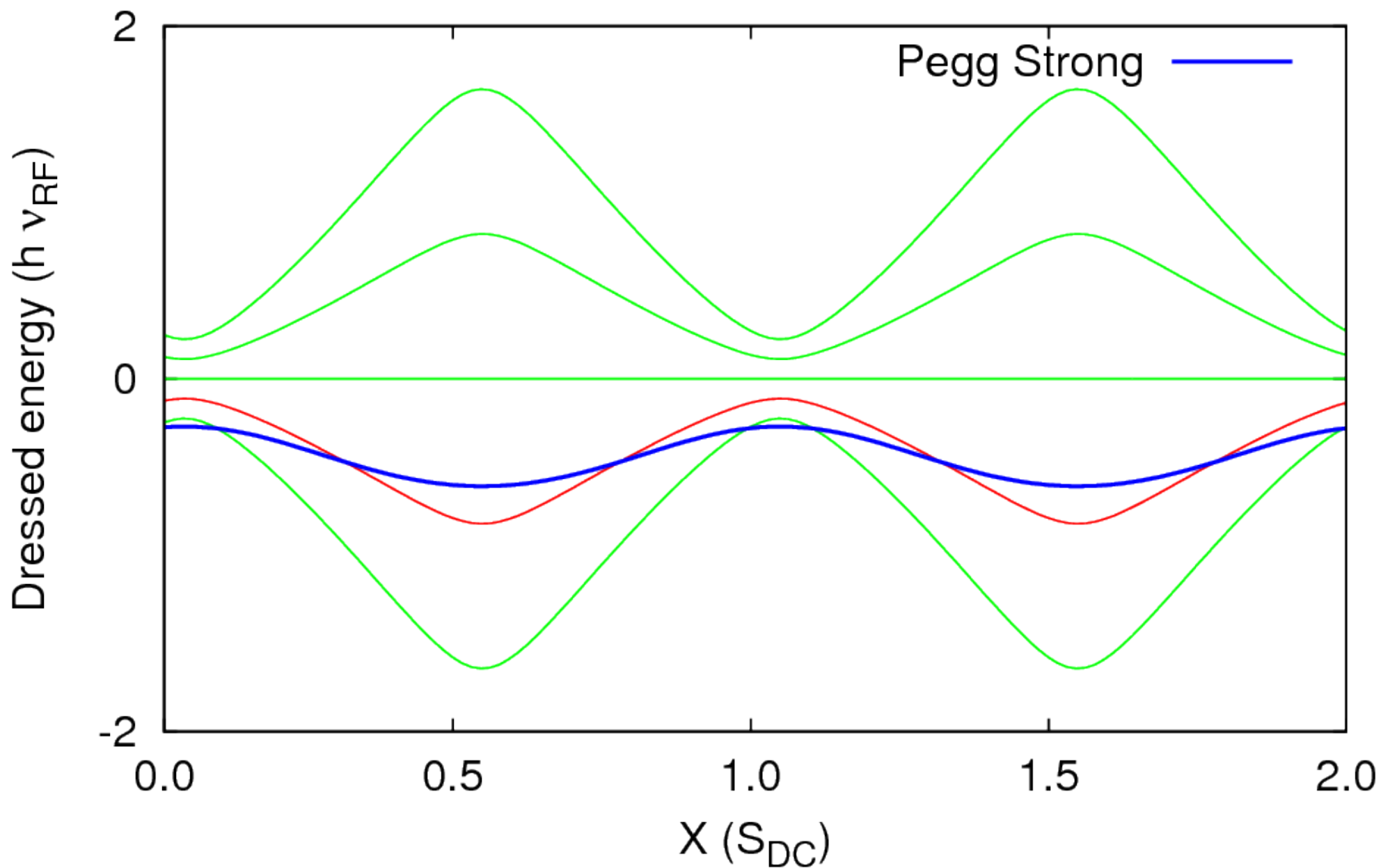
# Resonant driving, strong RF



Resonant driving, strong RF

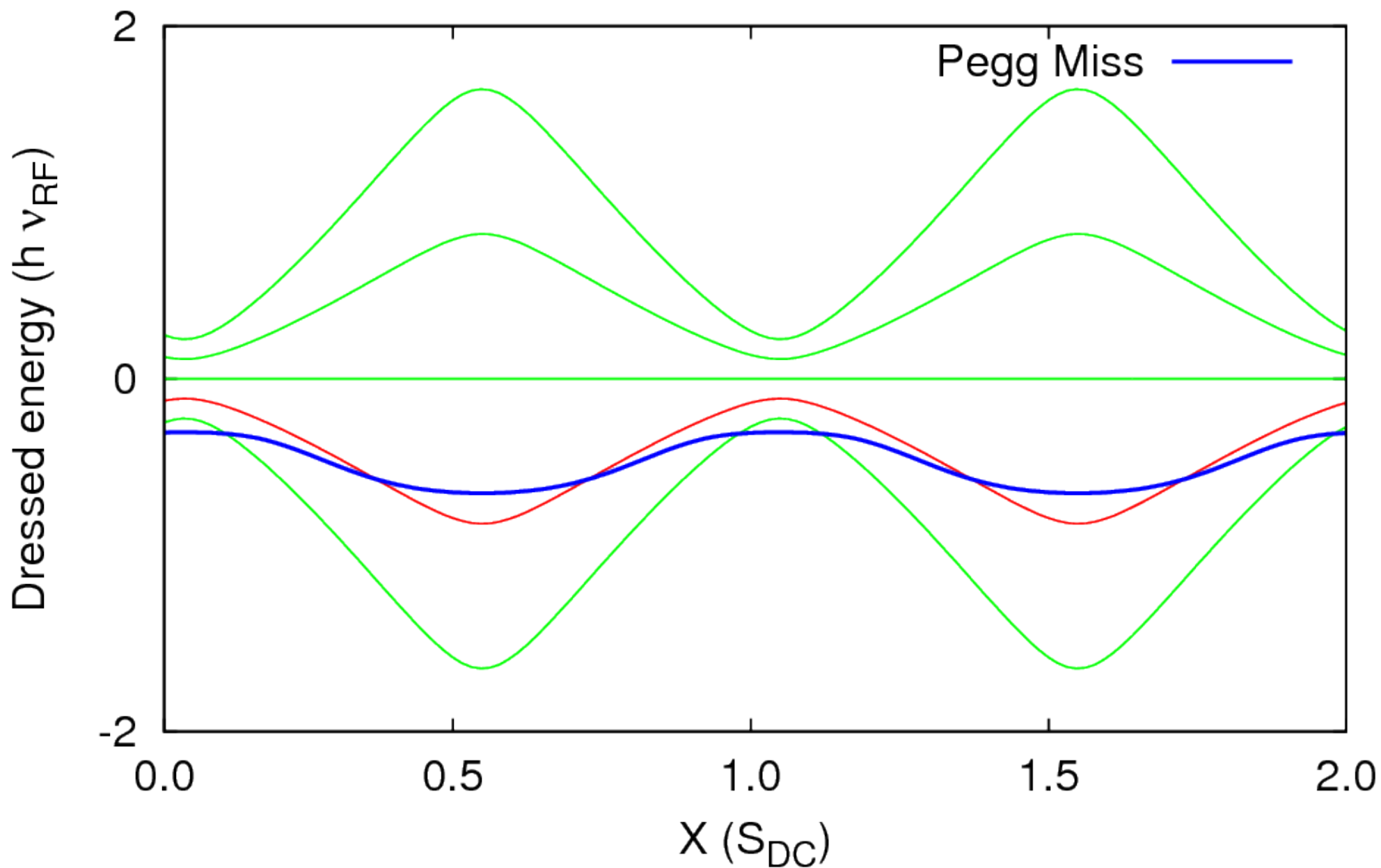


# Resonant driving, strong RF

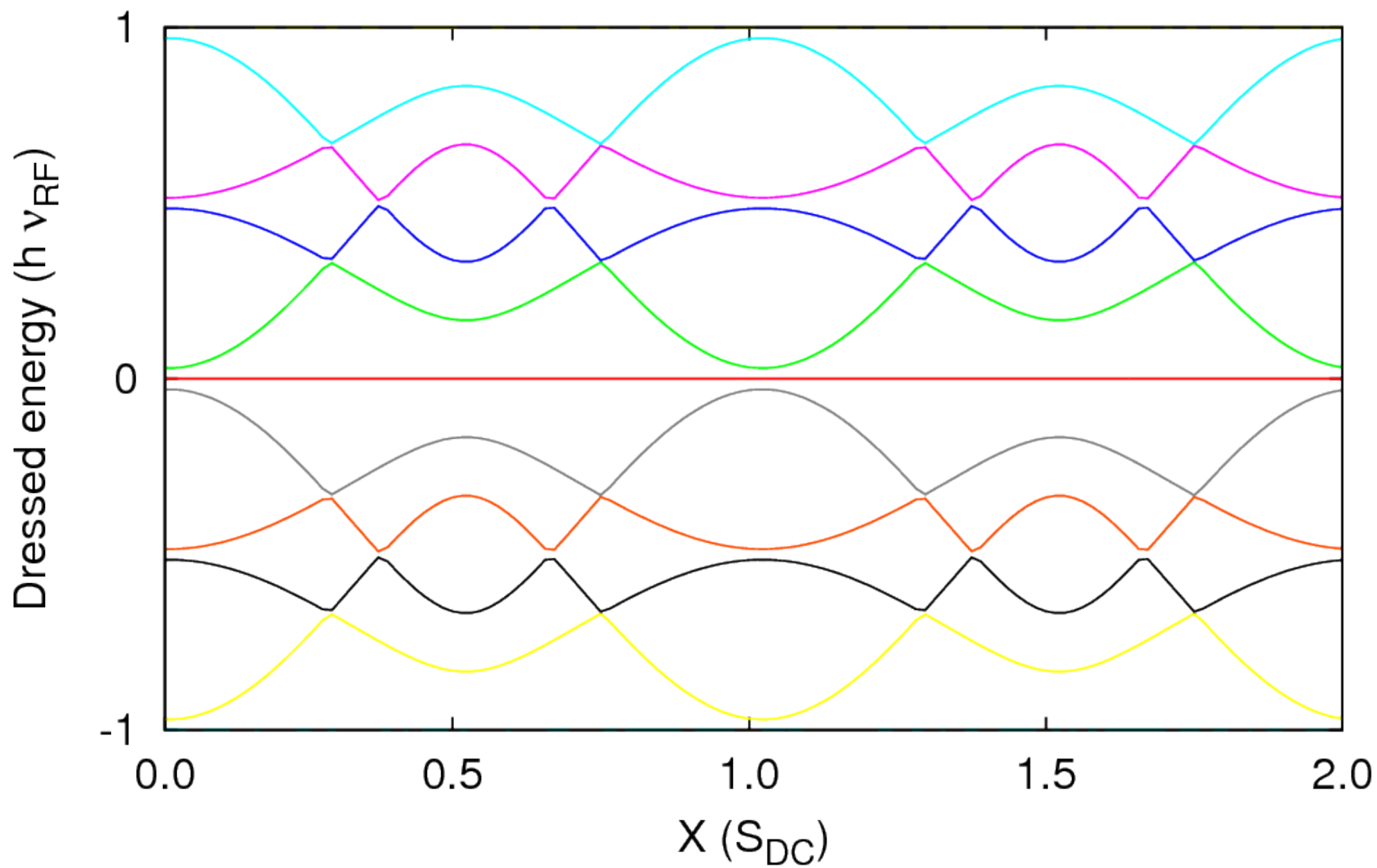




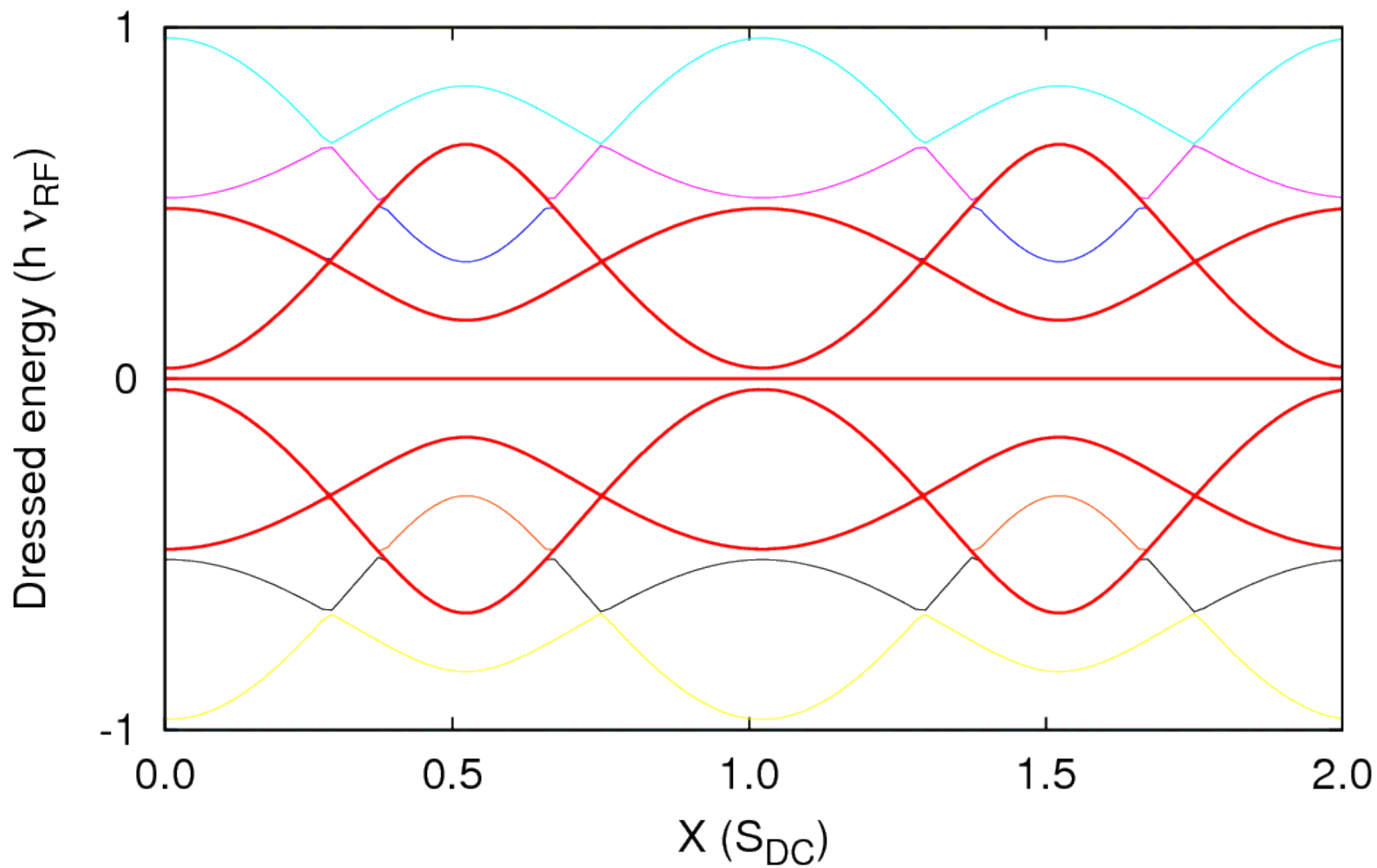
# Resonant driving, strong RF



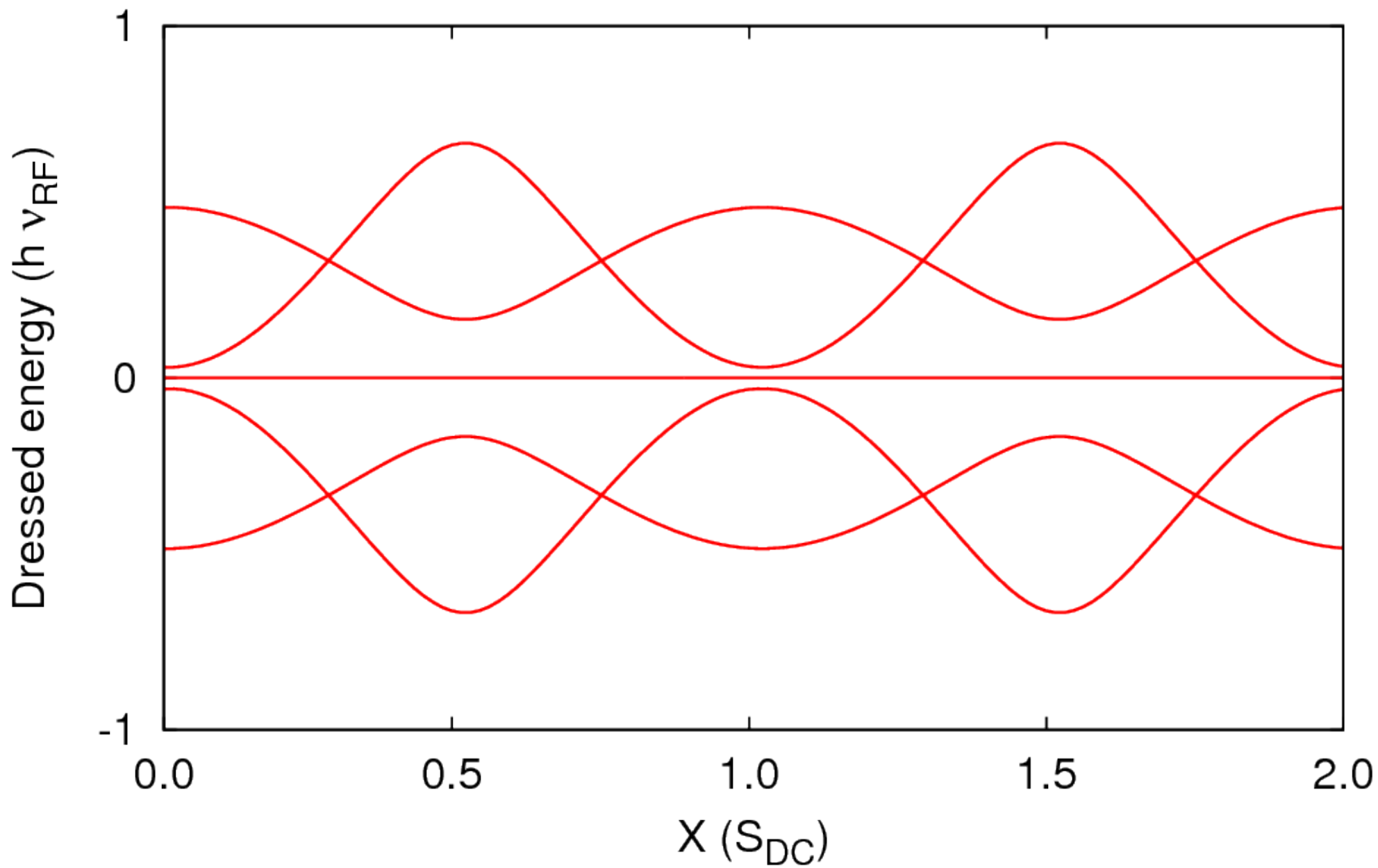
# Resonant driving, Weak RF



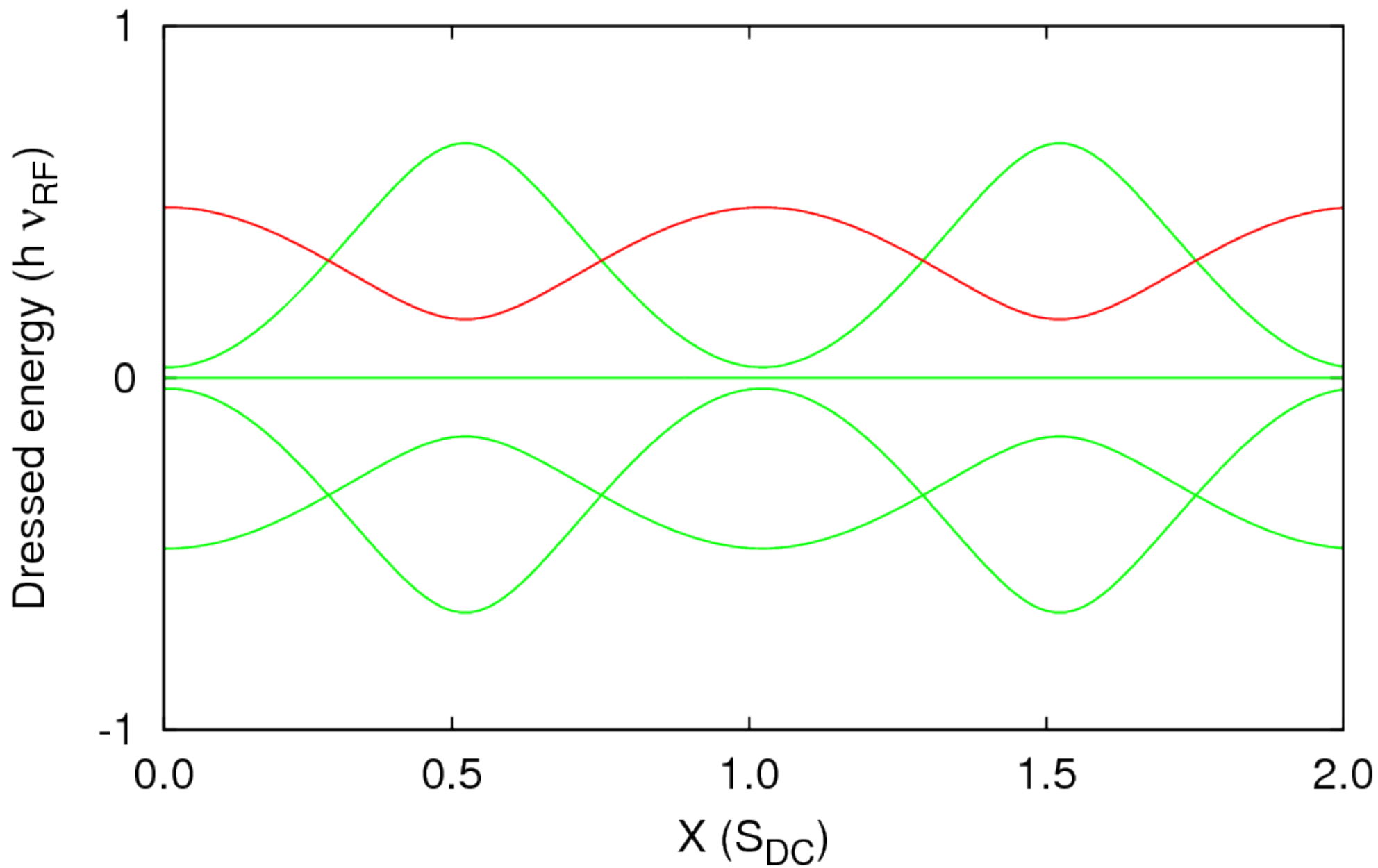
# Resonant driving, Weak RF



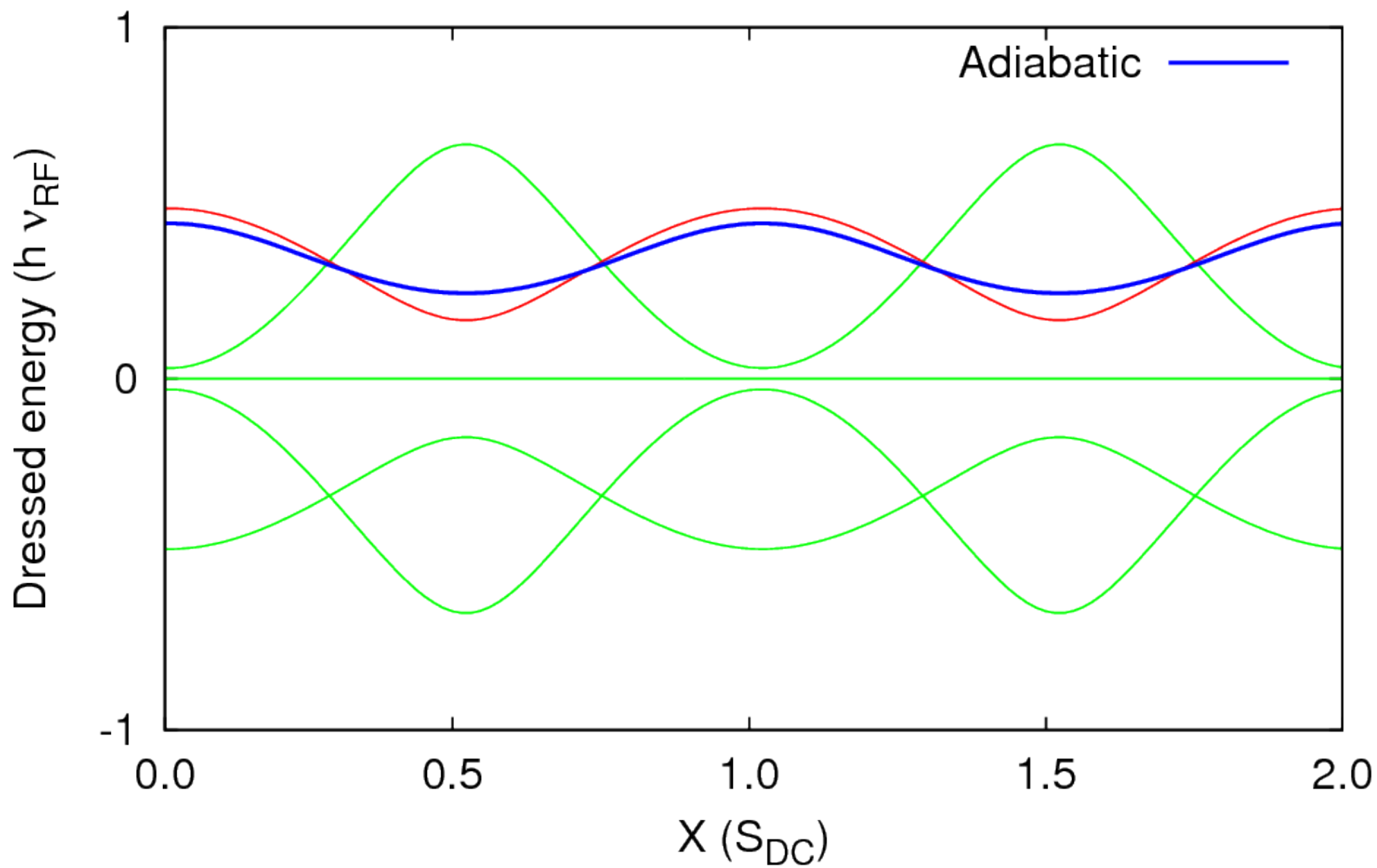
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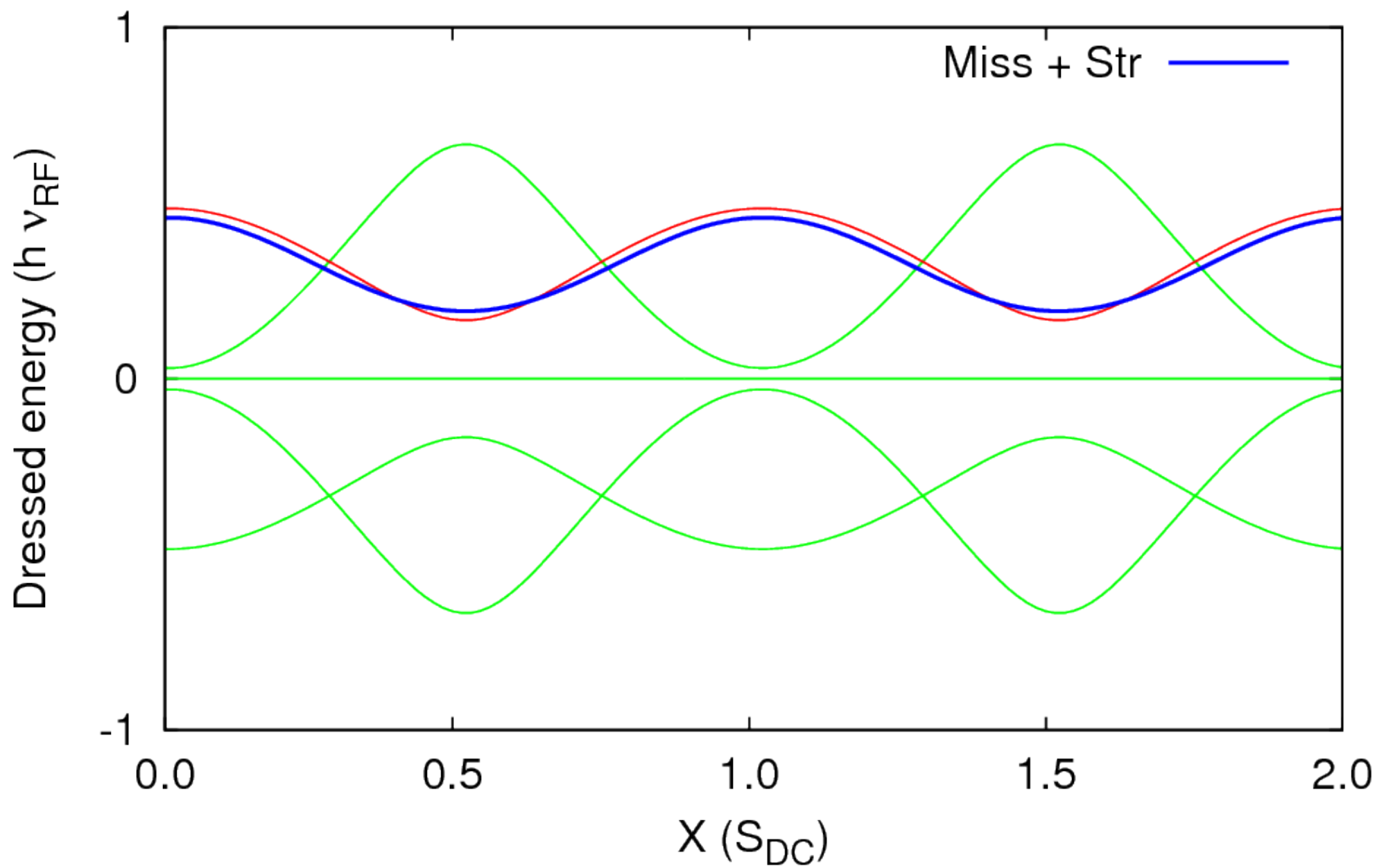
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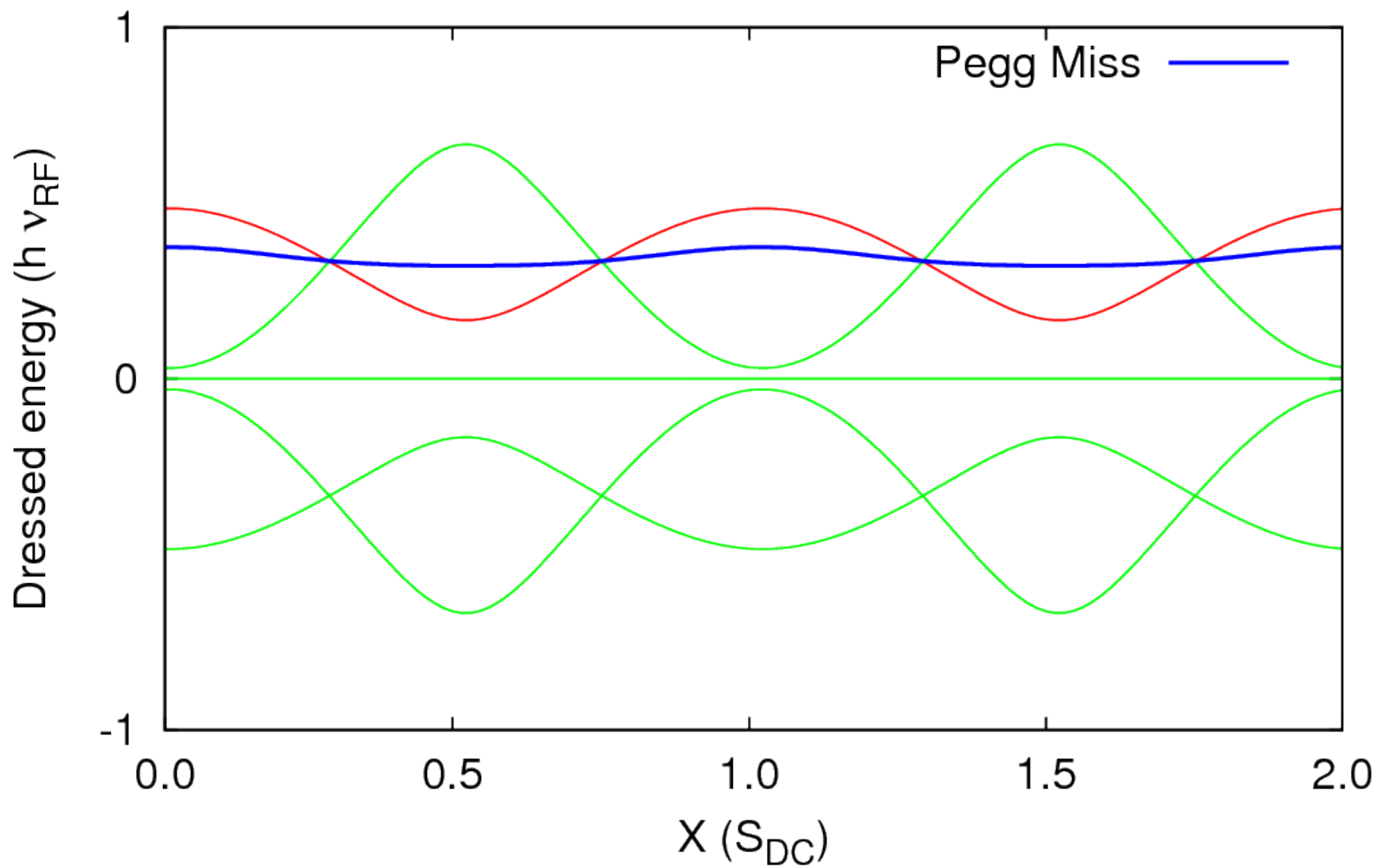
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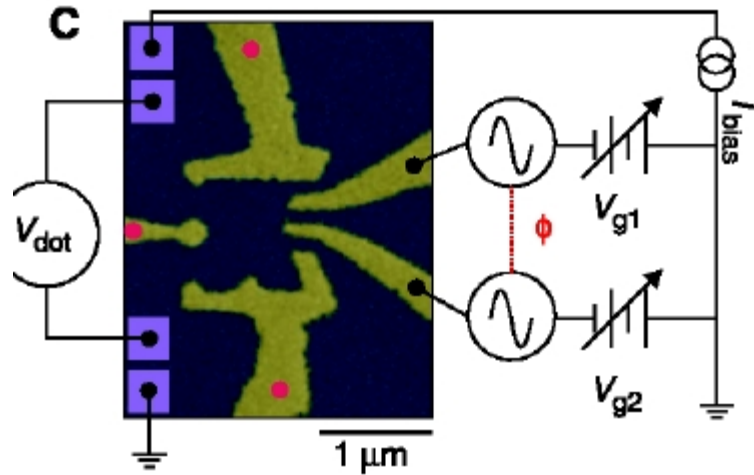
# Resonant driving, Weak RF



# Resonant driving, Weak RF







Swites, *et al.*, Science **83**, 1905 (1999)

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O. Entin-Wohlman, A. Aharony, and Y. Levinson, Phys. Rev. B 65, 195411 (2002).