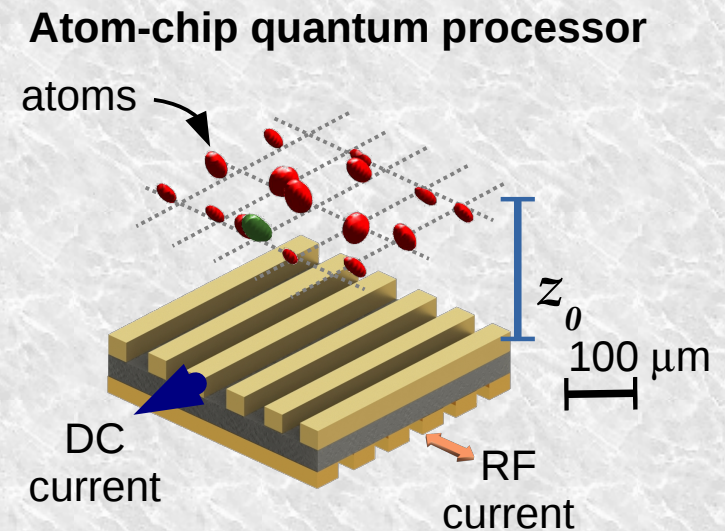


Spatial and internal control of atomic ensembles with harmonic drivings

German A. Sinuco-Leon
Department of Physics and Astronomy
University of Sussex



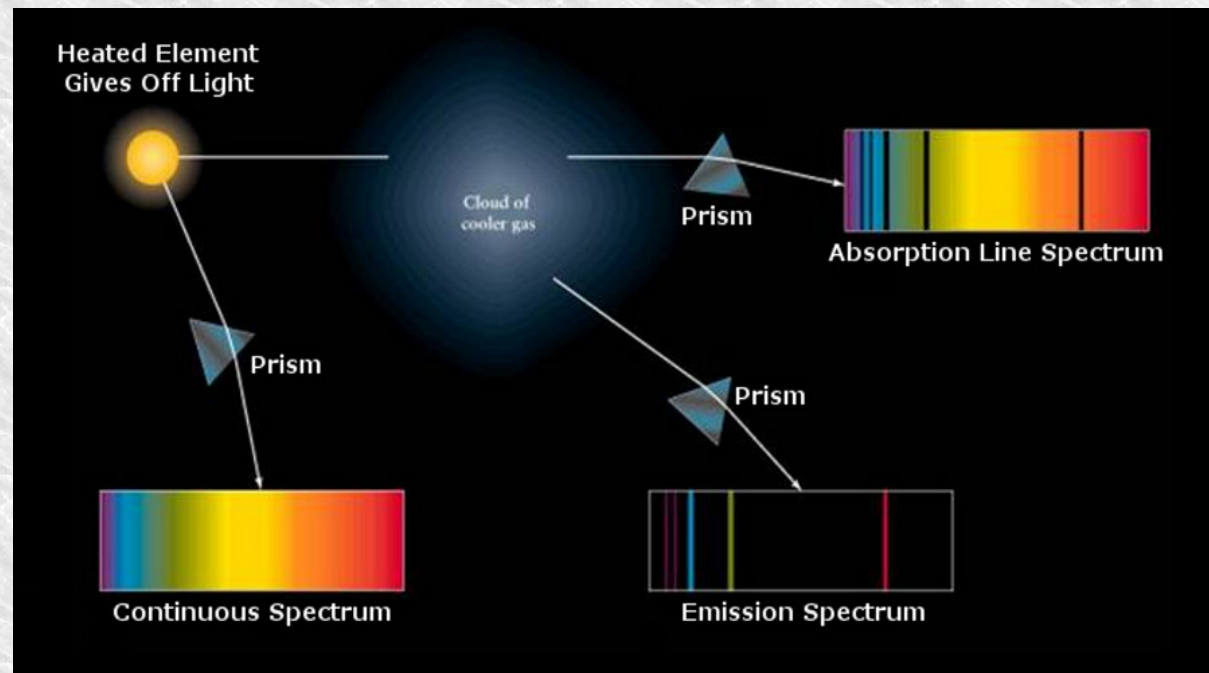
UCL, AMOP seminar, 7th November 2019

Outline

- ✓ Some driven quantum systems
- ✓ Controlling ^{87}Rb with low-frequency electromagnetic fields
- ✓ Time-evolution operator of driven quantum systems
- ✓ Outlook

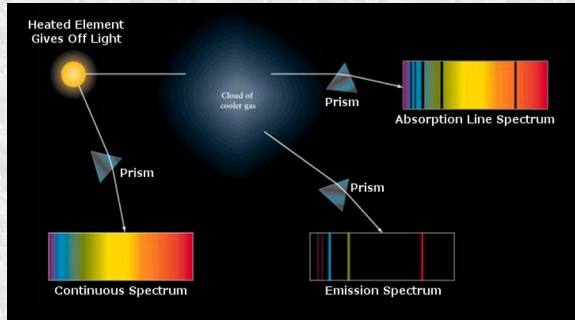
Driving physical systems

If you want to know what it is, just shake it



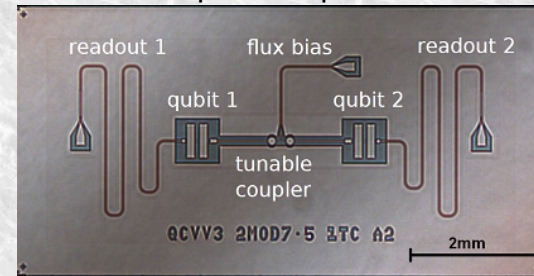
Driving physical systems

Investigate/identify materials



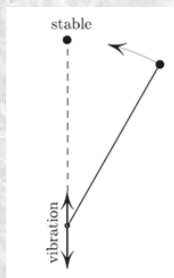
Quantum state control

IBM quantum processor

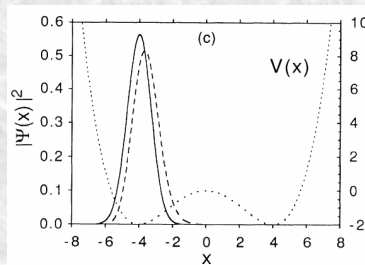


Phys. Rev. Applied 6, 064007 (2016)

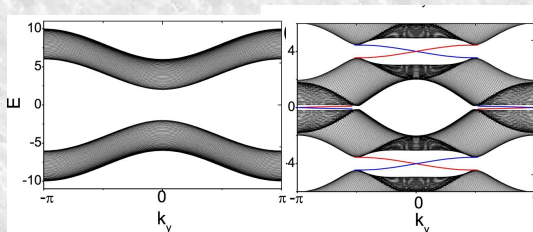
Modify the properties of a system



Kapitza pendulum

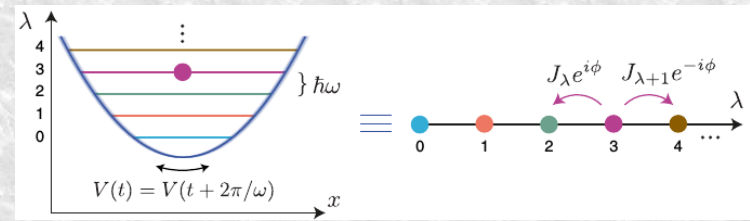


Coherent destruction of tunnelling
Grossmann, *et al.* PRL 1991

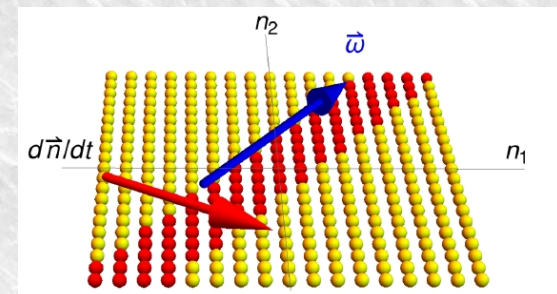


Floquet Engineering

Time-domain quantum simulation



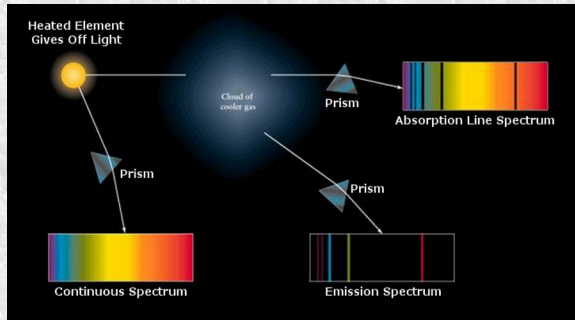
Price, Ozama and Goldman, PRA 95, 023607 (2017)



Martin, Refael and Halperin, PRX 7, 041008 (2017)

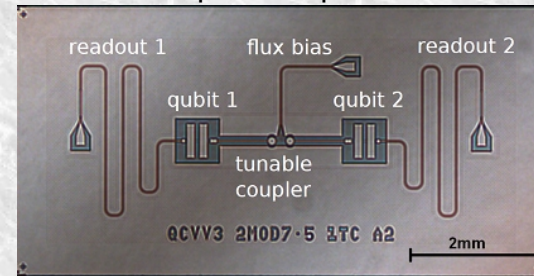
Driving physical systems

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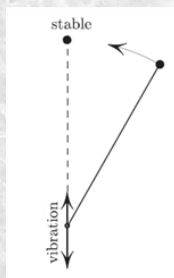
Quantum state control

IBM quantum processor

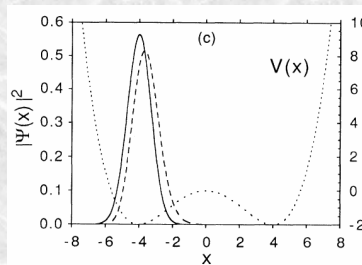


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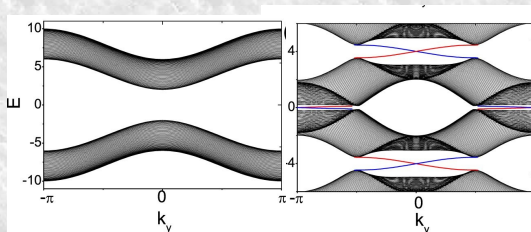
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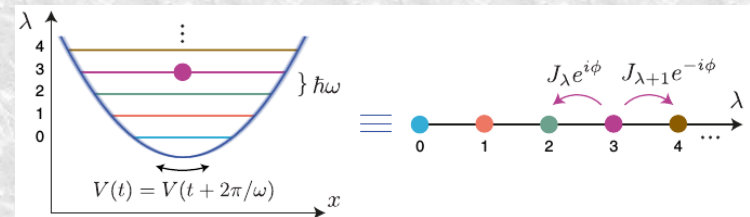


Coherent destruction of tunnelling
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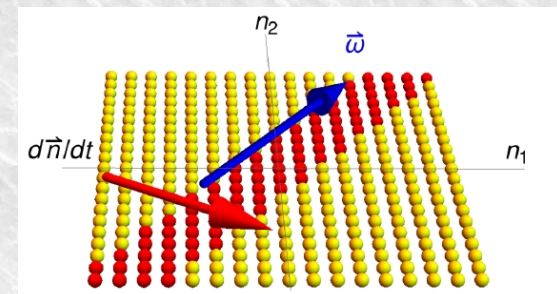


Floquet Engineering

Time-domain quantum simulation

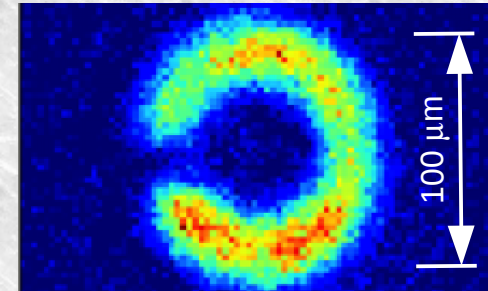
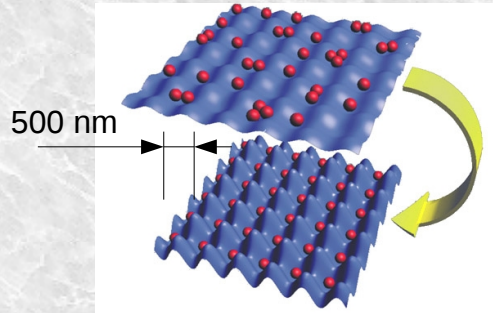
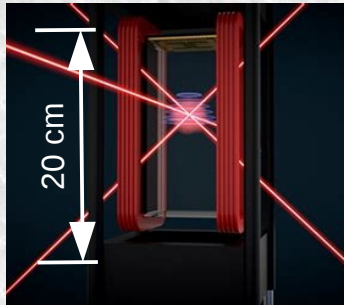


Price, Ozama and Goldman, PRA 95, 023607 (2017)



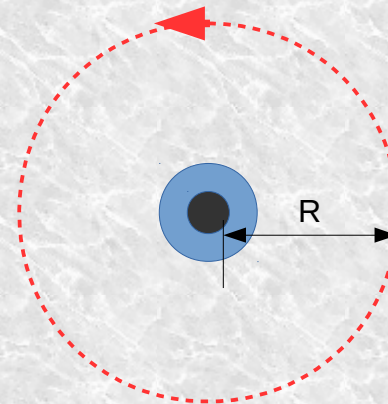
Martin, Refael and Halperin, PRX 7, 041008 (2017)

Controlling atomic ensembles

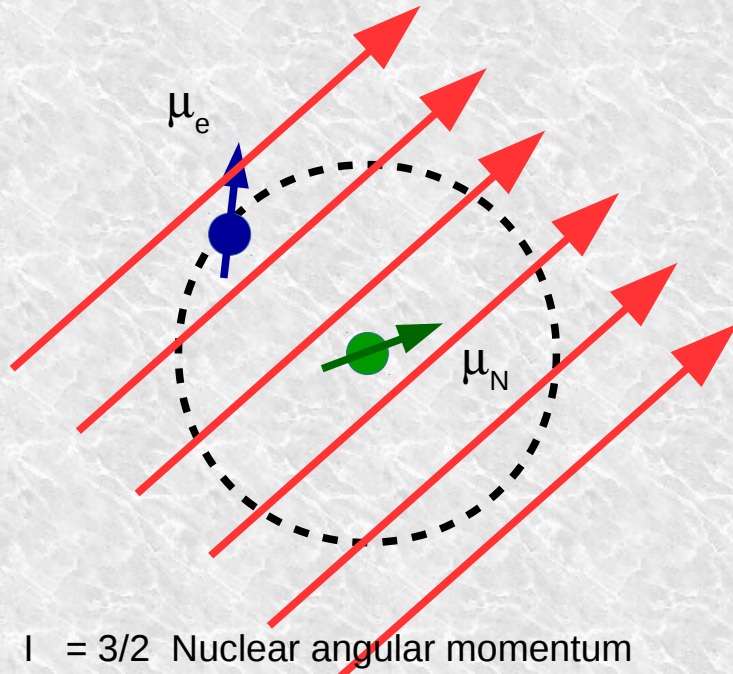


- Laser cooling produce atomic ensembles with temperatures in the range between $1 \mu\text{k}$ – 1nK
- At such temperature, the kinetic energy is comparable with the interaction energy with electric and magnetic field produced by nearby sources.
- At $T = 1 \mu\text{K}$, the magnetic field produced by a current of 1 mA can captures an atom of Rb87 in a circular orbit of radius $R = 200 \mu\text{m}$

Current = 1 mA
Temperature = $1 \mu\text{K}$
Speed = 5 mm/s
Radius = $200 \mu\text{m}$



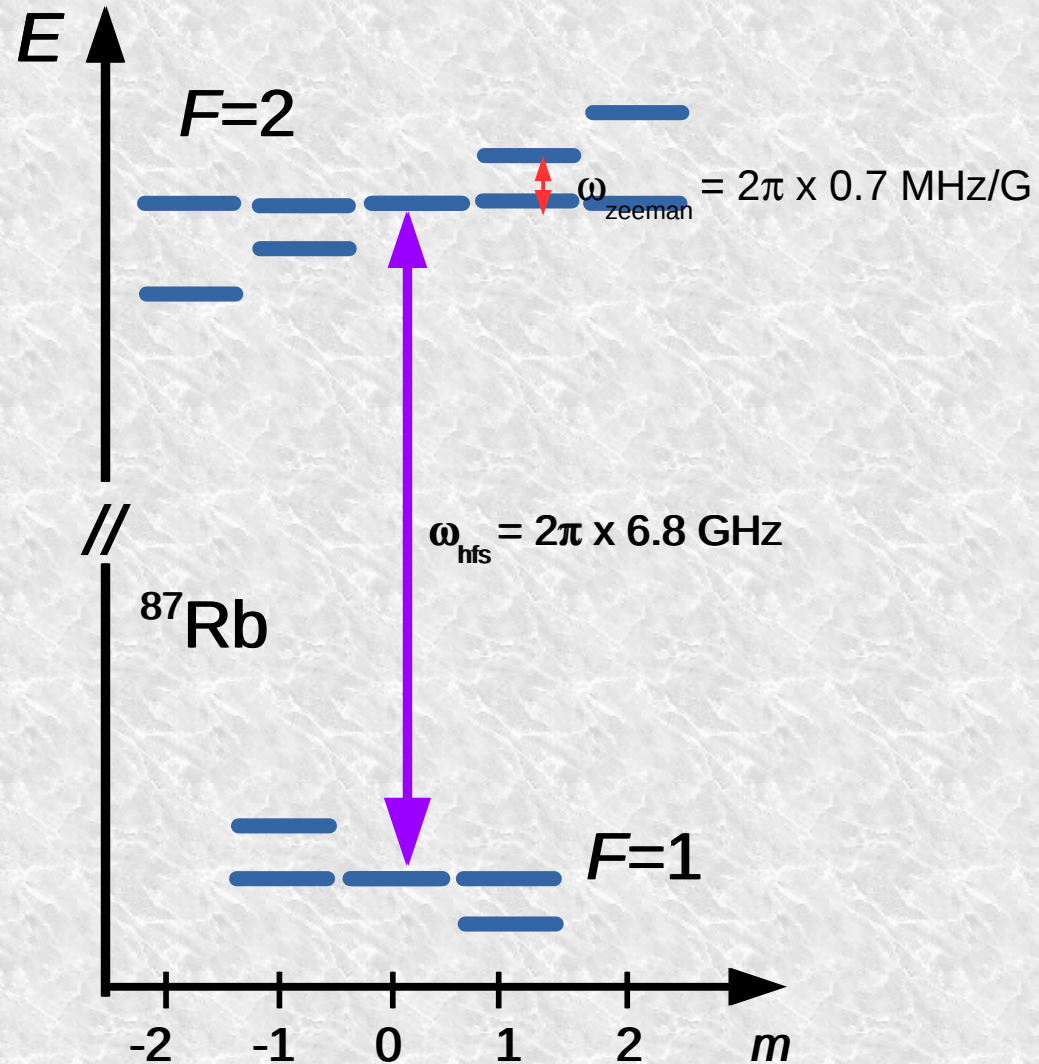
87Rb



- $I = 3/2$ Nuclear angular momentum
- $S = 1/2$ e⁻ spin angular momentum
- $J = 0$ e⁻ orbital angular momentum

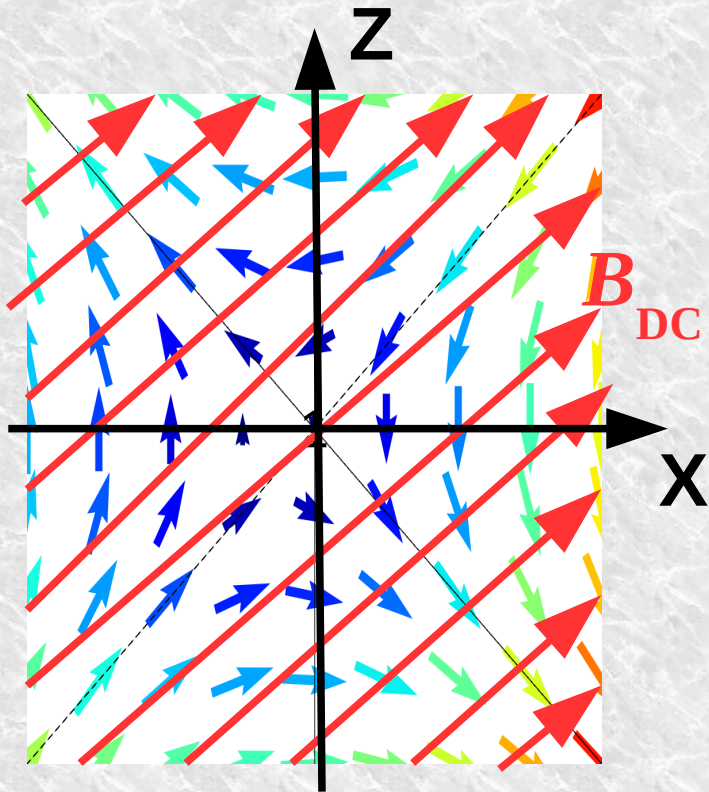
As a result, the electronic ground state manifold can be controlled using a combination of microwave and radiofrequency electromagnetic fields.

$$H = \frac{P^2}{2m} + A \hat{J} \cdot \hat{I} + \mu_B \mathbf{B} \cdot (g_J \hat{J} + g_I \hat{I})$$



Consider an alkali atom slowly moving through a region with inhomogeneous static and a radiofrequency field:

$$H = \frac{P^2}{2m} + \mu_B g_F \mathbf{B}_{DC} \cdot \hat{\mathbf{F}} + \mu_B g_F \mathbf{B}_{RF} \cdot \hat{\mathbf{F}} \cos \omega t$$



1. Perform a geometrical rotation to orientate the static field along the z axis.
2. Move to a rotating frame of reference where it is safe to ignore time-dependent terms:

$$U(\mathbf{r}) = \exp\left(-i \frac{g_F}{|g_F|} \omega t \hat{\mathbf{F}}_z\right) R_{DC}(\mathbf{r})$$

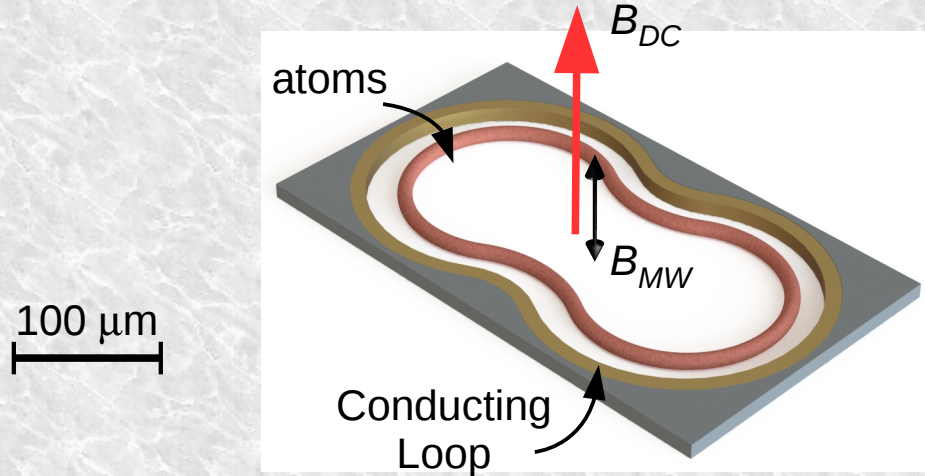
- Rotating wave approximation
- Adiabatic approximation.

$$H \approx \frac{P^2}{2m} + (\mu_B g_F B_{DC} - \hbar \omega) \hat{\mathbf{F}}_z + \frac{\mu_B g_F B_{RF}}{2} \hat{\mathbf{F}}_x$$

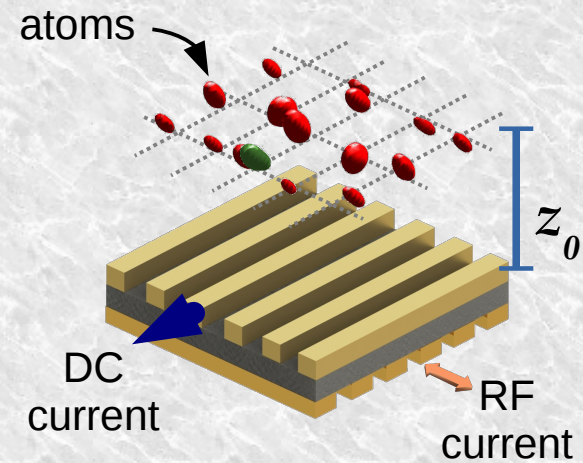
$$H \approx \frac{P^2}{2m} + \sqrt{(\mu_B g_F B_{DC} - \hbar \omega)^2 + \left(\frac{\mu_B g_F B_{RF}}{2}\right)^2} \hat{\mathbf{F}}_z$$

Controlling 87Rb

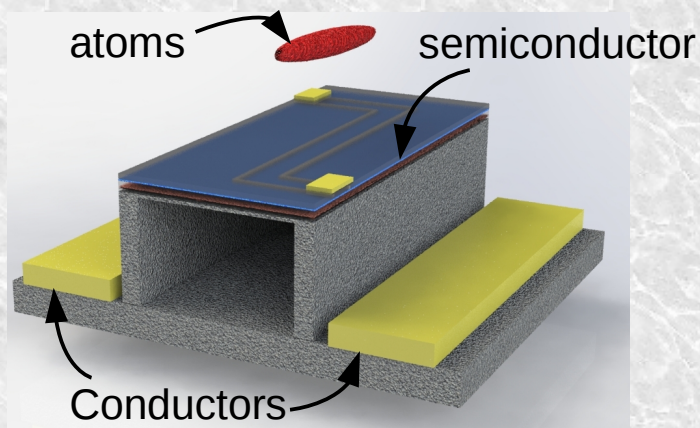
Atom-chip for rotational sensing
Nat. Comm. 2014.



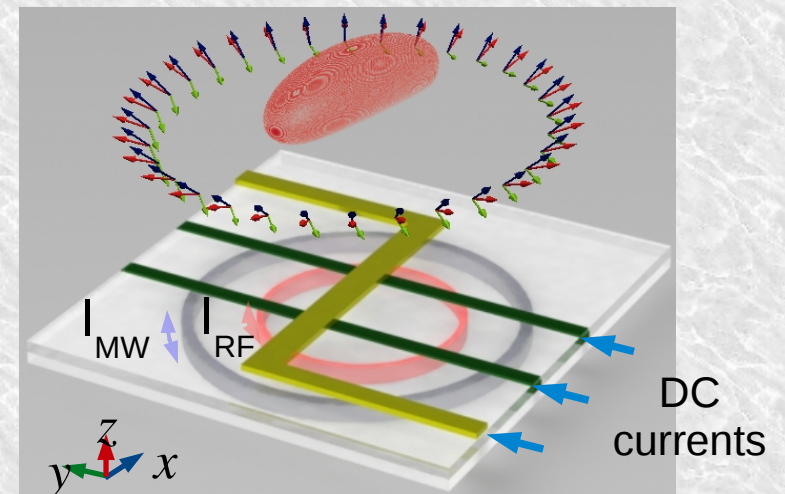
Atom-chip for Quantum Sim. and IP
NJP 2015 and NJP. 2016.



Hybrid atom-chip technology
JMO 2018.



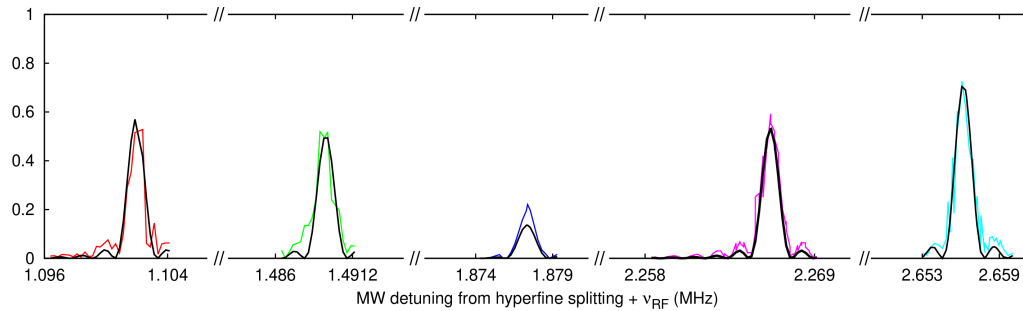
RF + MW dressing
In prep., 2019.



Controlling 87Rb

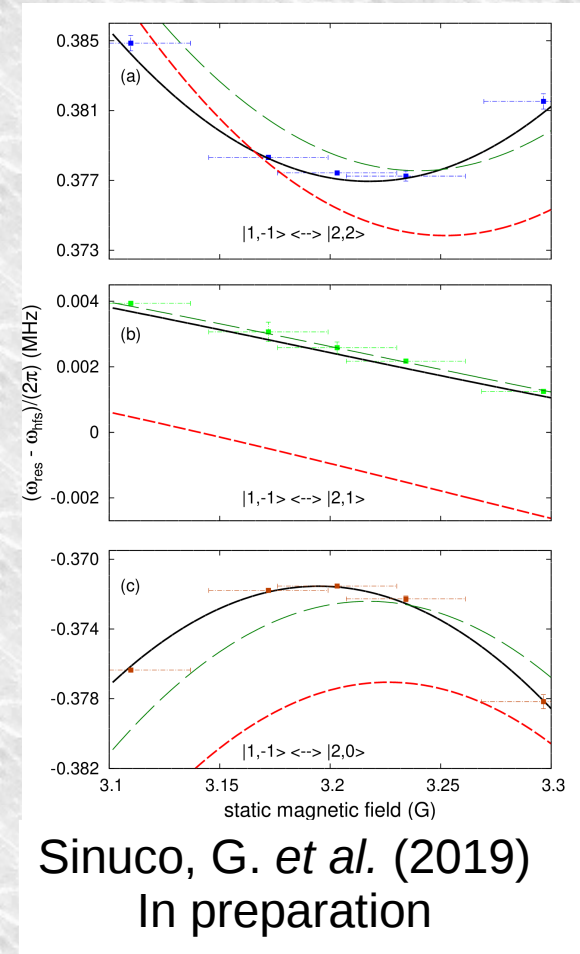
Protection of microwave qubits with radio-frequency dressing

Microwave spectroscopy of radio-frequency dressed 87Rb



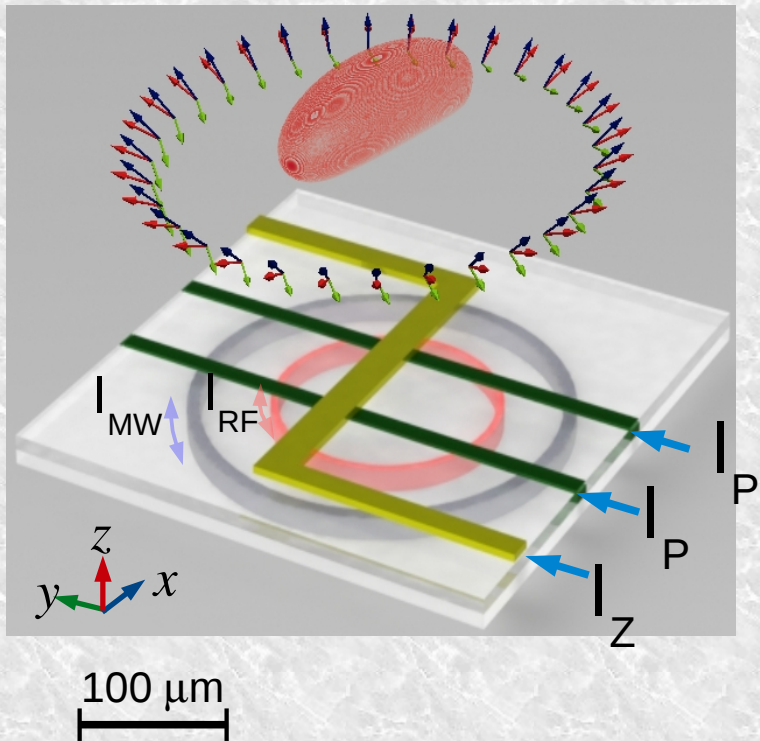
Sinuco, G. *et al.* ArXiv 1904.12073

With B. Garraway (Sussex), W. von Klitzing (Crete)
and T. Fernolhcz (Nottingham)



Sinuco, G. *et al.* (2019)
In preparation

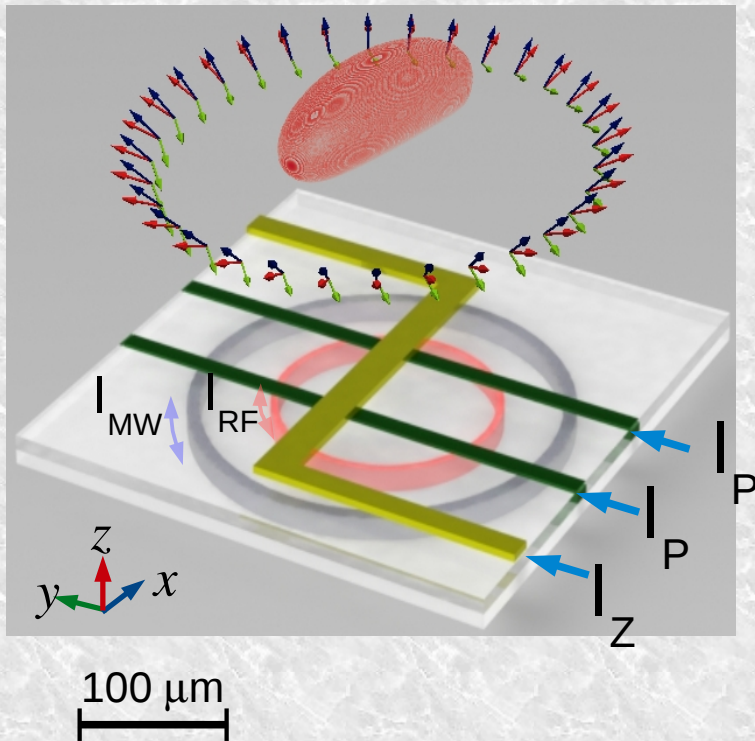
Controlling ^{87}Rb



To do:

- ✓ Experimental bounds of the parameter space
- ✓ Define the problem and geometry
- ✓ Electromagnetic simulator
- ✓ Calculate the effective energy potential landscape
- ✓ Estimate non-adiabatic effects
- ✓ Evaluate near surface effects:
 - Johnson-noise
 - Casimir-Polder attraction

Controlling ^{87}Rb



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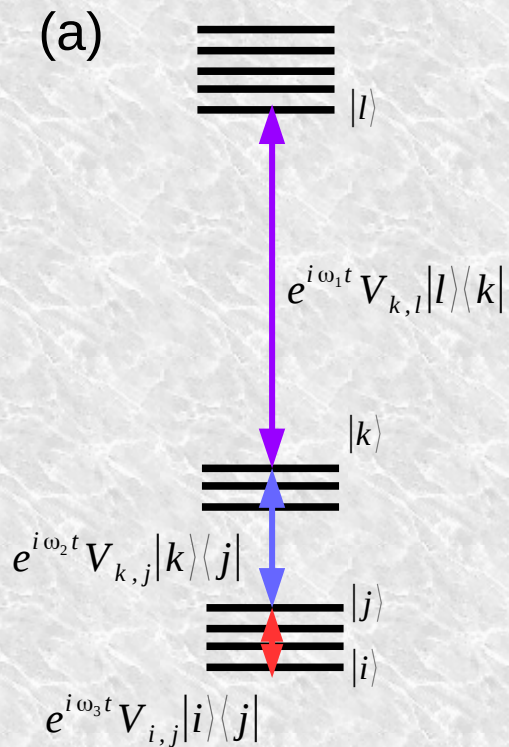
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- ✓ Estimate non-adiabatic effects
- ✓ Evaluate near surface effects:
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- Beyond Rotating wave approximation
- Polychromatic driving: RF + MW

$$H \approx \frac{\mathbf{P}^2}{2m} + \sqrt{(\mu_B g_F B_{DC} - \hbar \omega)^2 + \left(\frac{\mu_B g_F B_{RF}}{2}\right)^2} \hat{F}_z$$

Driven Quantum Systems

Generic energy level structure of a quantum system. The arrows represent harmonic couplings of different frequencies.

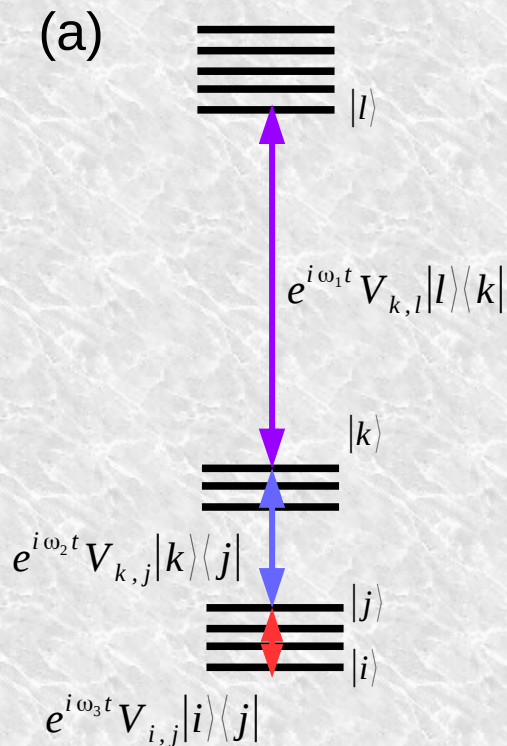


Driven Quantum Systems

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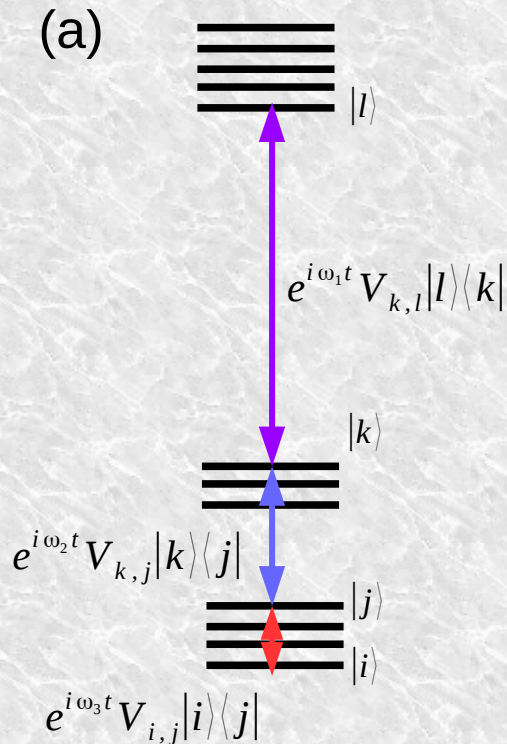
We consider the Hamiltonian of a quantum system driven by several harmonic fields:

$$H = \sum_i E_i |i\rangle\langle i| + \sum_l \sum_n \sum_{i,j} V_{ij} e^{n\omega_l t} |i\rangle\langle j| + h.c.$$



Driven Quantum Systems

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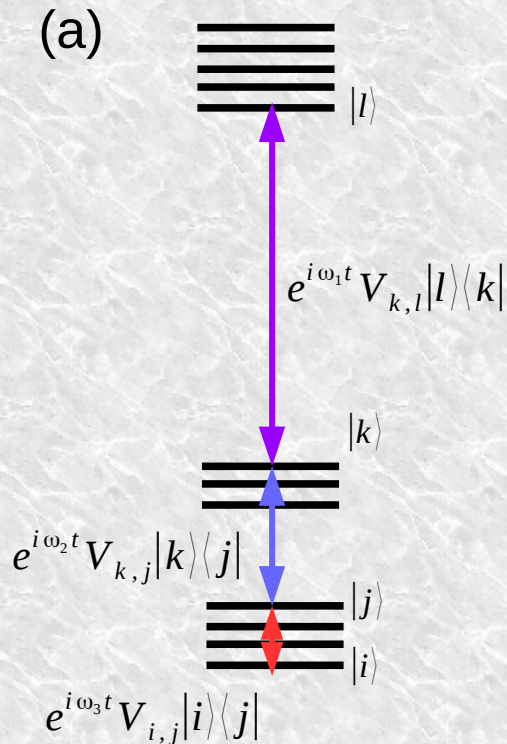
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To find the time-evolution operator, we build a unitary transformation that takes the Hamiltonian to a time-independent and diagonal form, i.e.:

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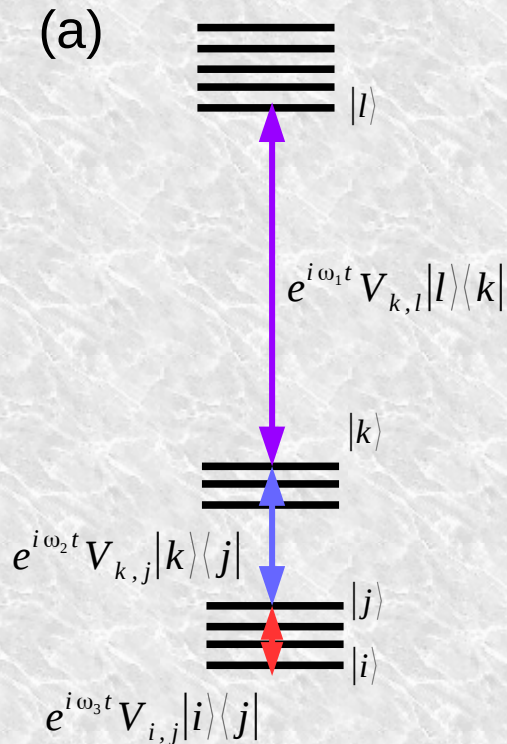
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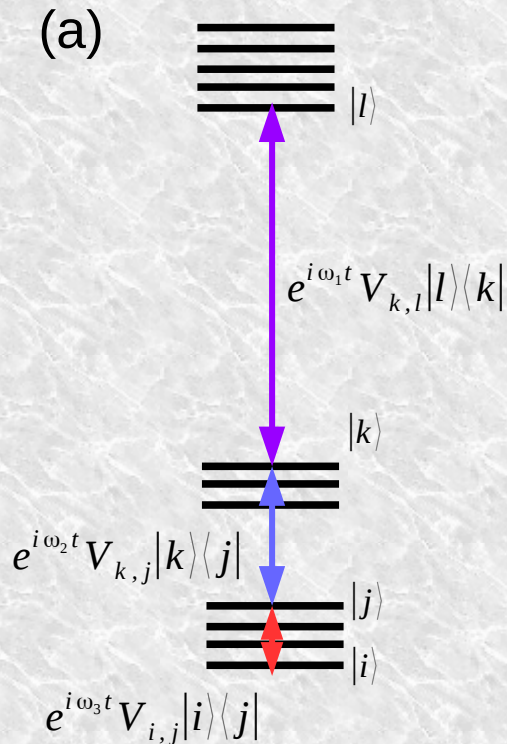
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As a function of position, these values are the State-dependent potential energy landscape

Driven Quantum Systems

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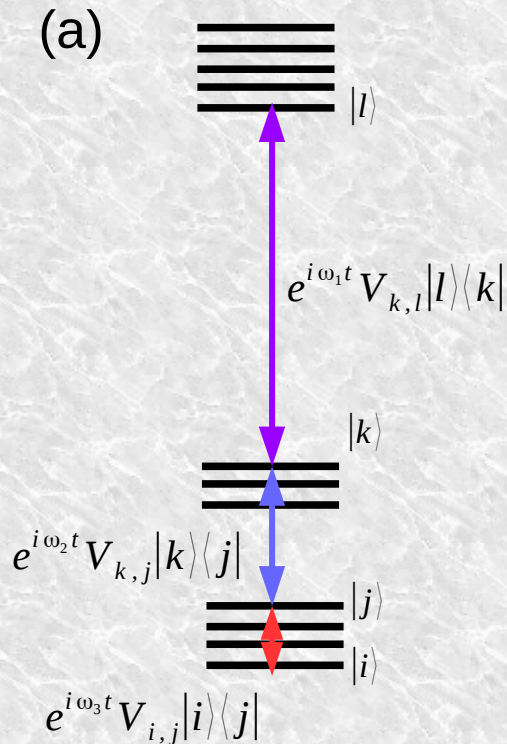
$$\hat{H} = \hat{U}_F^\dagger \hat{H} \hat{U}_F - i\hbar \hat{U}_F^\dagger \partial_t \hat{U}_F = \sum_{\lambda} \lambda |\lambda\rangle\langle\lambda|$$

The harmonic dependence of the driving let us find this unitary transformation using the Fourier decomposition:

$$\hat{U}_F(t) = \sum_{i\lambda} e^{i\vec{n}\cdot\vec{\omega}t} u_{i\lambda}^{\vec{n}} |i\rangle\langle\lambda|$$

Driven Quantum Systems

Generic energy level structure of a quantum system. The arrows represent harmonic couplings of different frequencies.



We consider the Hamiltonian of a quantum system driven by several harmonic fields:

$$H = \sum_{\vec{n}} \hat{H}_{\vec{n}} e^{i\vec{n}\cdot\vec{\omega}t}$$

This procedure defines the micromotion operator:

$$\hat{U}_F(t) = \sum_{i\lambda} e^{i\vec{n}\cdot\vec{\omega}t} u_{i\lambda}^{\vec{n}} |i\rangle\langle\lambda|$$

and the basis of dressed states:

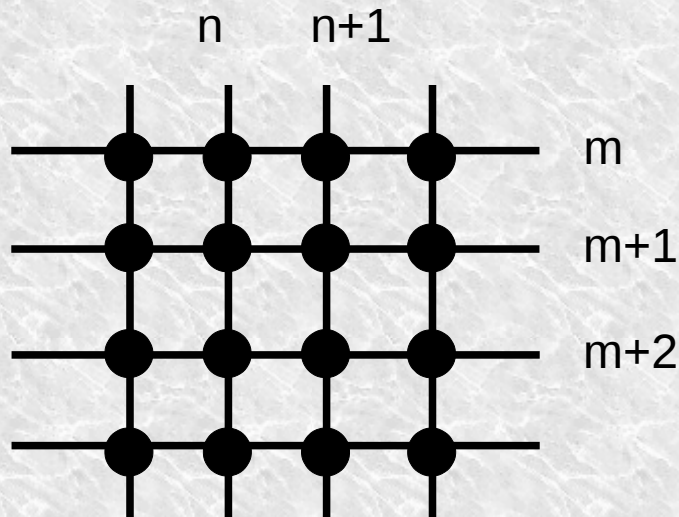
$$\{|\lambda\rangle, |\lambda\rangle\}$$

As a time-dependent superposition of bare states

$$U(t', t) = U_F(t') e^{i\hat{H}(t'-t)} U_F^\dagger(t)$$

U(t',t) – Floquet band engineering

Shining light on conventional insulators produces a system whose effective energy bands have a non-trivial topology. (Linder, Nat. Phys. 7, 490 (2011))



$$H(t) = \sum_{n,m} J_x(t) a_{n,m}^\dagger a_{n+1,m} + h.c. \\ + J_y(t) \exp(i\alpha n) a_{n,m}^\dagger a_{n,m+1} + h.c.$$

$$H = \sum_m \hat{H}_m e^{im\omega t}$$

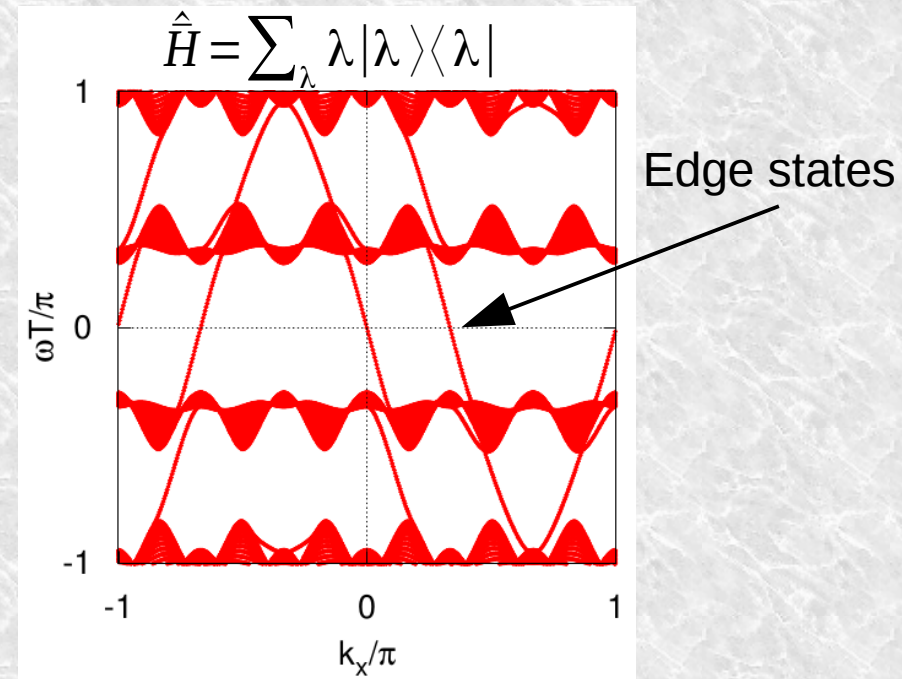
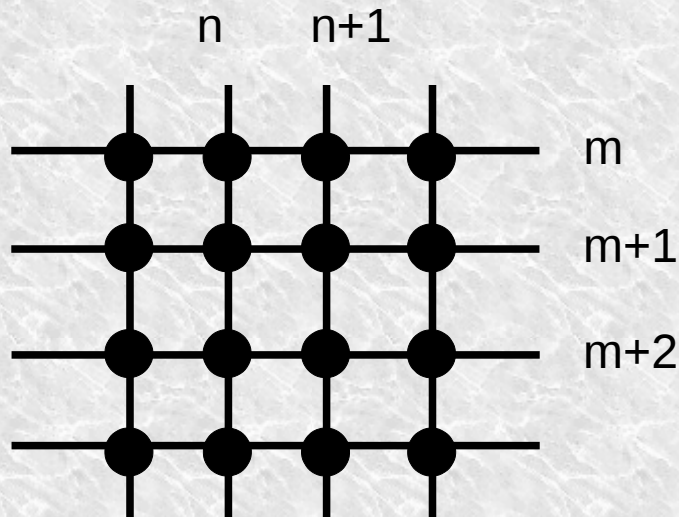
$$U(t',t) = U_F(t') e^{i\hat{H}(t'-t)} U_F^\dagger(t)$$

$$\hat{H} = H_0 + \sum_m \frac{1}{m\hbar\omega} [\hat{H}_m, \hat{H}_m^\dagger] + \dots$$

Anisimovas, et al., ArXiv: 1504.03583

U(t',t) – Floquet band engineering

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- The time-evolution operator determines the bulk-edge correspondence. Roy and Harper, arxiv 1603.06944 (2016)

- The micromotion operator determines the time-scale of heating effects (arxiv 1508.0579)

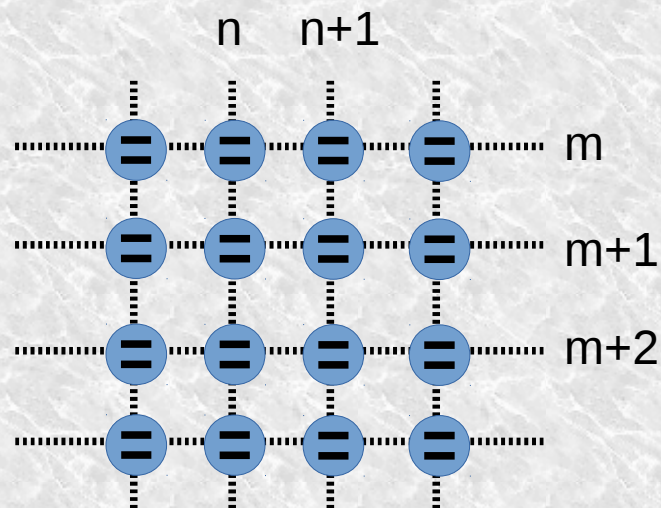
U(t',t) – Time-domain quantum simulations of lattice models



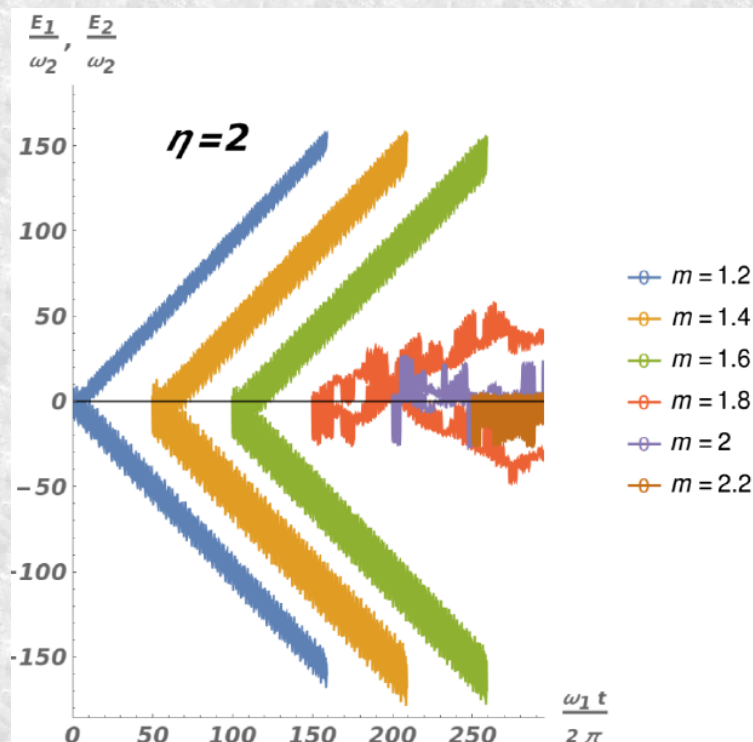
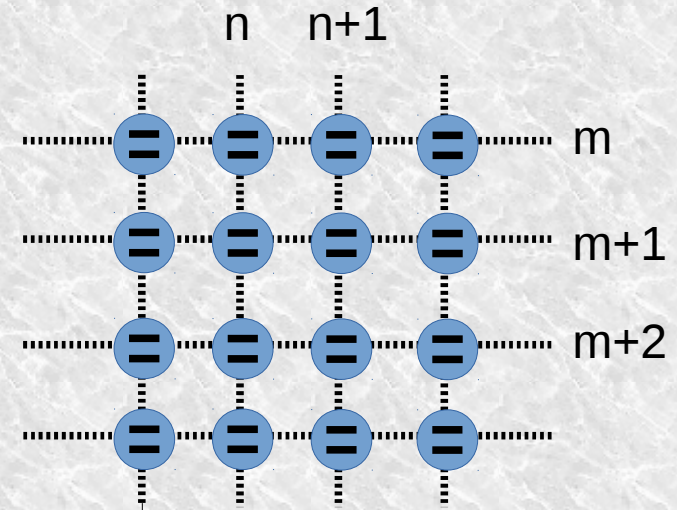
$$H(t) = m S_z + v_1 \sin(\omega_1 t + \varphi_1) S_x - b_1 \cos(\omega_1 t + \varphi_1) S_z + v_2 \sin(\omega_2 t + \varphi_2) S_y - b_2 \cos(\omega_2 t + \varphi_2) S_z$$



$$H = m S_z + \vec{n} \cdot \vec{\omega} + (v_1 e^{i\varphi_1} S_x - b_1 e^{i\varphi_1} S_z) \delta_{n,n+1} - (v_1 e^{-i\varphi_1} S_x + b_1 e^{-i\varphi_1} S_z) \delta_{n,n-1} + (v_2 e^{i\varphi_2} S_y - b_2 e^{i\varphi_2} S_z) \delta_{m,m+1} - (v_2 e^{-i\varphi_2} S_x + b_1 e^{-i\varphi_2} S_z) \delta_{m,m-1}$$



$U(t',t)$ – Time-domain quantum simulations of lattice models



Energy absorption from drives 1 and 2, as functions of time for different m 's. Strong and red-detuned driving.

Time-evolution operator

"The essence of our theory is that by using the latter interpretation ... problems involving Hamiltonians periodic in time may be solved by methods applicable to time-independent Hamiltonians"

Jon Shirley, Phys. Rev. **138**, B979 (1969).

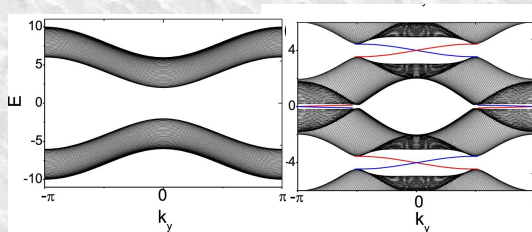
approaches developed for many-body lattice models

Floquet components of the micromotion operator

$$H = \sum_{\vec{n}} \hat{H}_{\vec{n}} e^{i\vec{n} \cdot \vec{\omega} t}$$

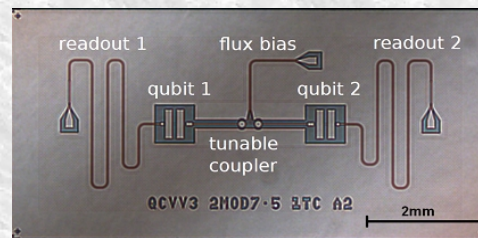
$$\hat{H} = \hat{U}_F^\dagger \hat{H} \hat{U}_F - i\hbar \hat{U}_F^\dagger \partial_t \hat{U}_F = \sum_{\lambda} \lambda |\lambda\rangle \langle \lambda|$$

Interplay between geometry, disorder, driving and interactions.



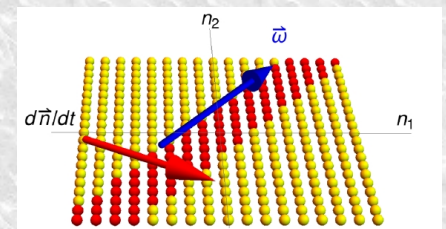
Dynamical Engineering

Quantum control



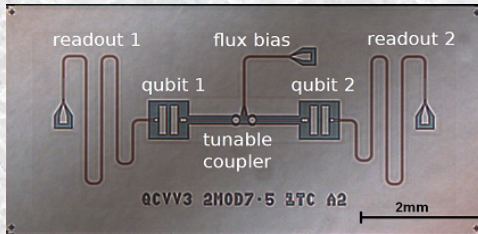
- ✓ Robust
- ✓ Fast

High-dimensional time-domain quantum simulations

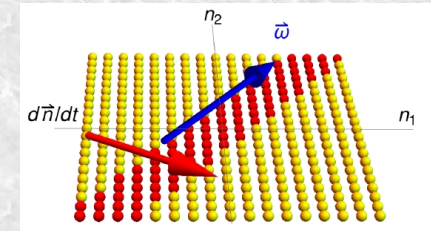


Outlook

Quantum control



Time-domain quantum simulations

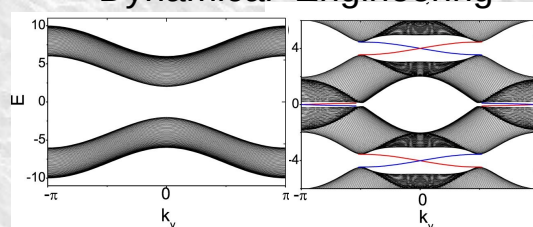


$$H = \sum_{\vec{n}} \hat{H}_{\vec{n}} e^{i\vec{n} \cdot \vec{\omega} t}$$

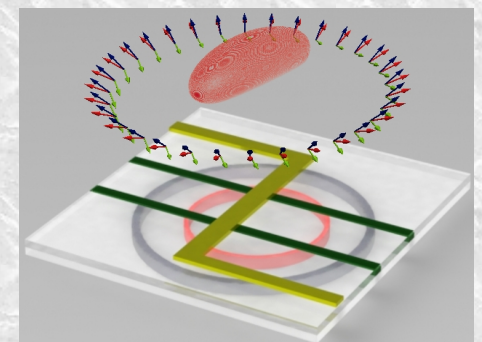
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$$\hat{U}_F(t) = \sum_{i\lambda} e^{i\vec{n} \cdot \vec{\omega} t} u_{i\lambda}^{\vec{n}} |i\rangle \langle \lambda|$$

Dynamical Engineering



Atomic Quantum technology



Renormalisation of the Floquet Hamiltonian

$$\hat{H} = \hat{U}_F^\dagger \hat{H} \hat{U}_F - i\hbar \hat{U}_F^\dagger \partial_t \hat{U}_F = \sum_{\lambda} \lambda |\lambda\rangle\langle\lambda|$$

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1D biased tight binding model



Renormalisation of the Floquet Hamiltonian

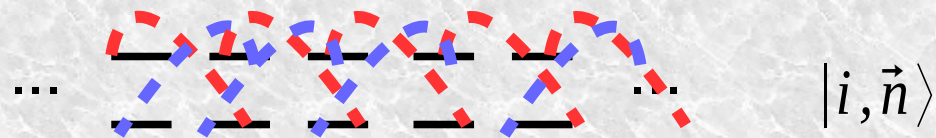
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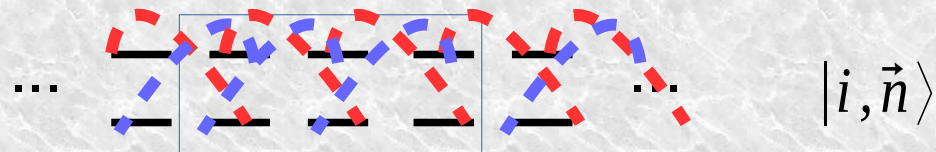
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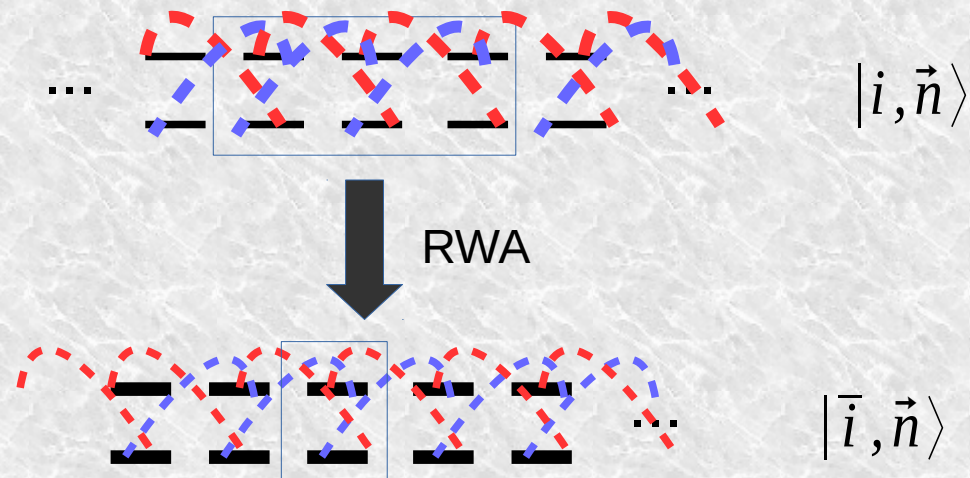
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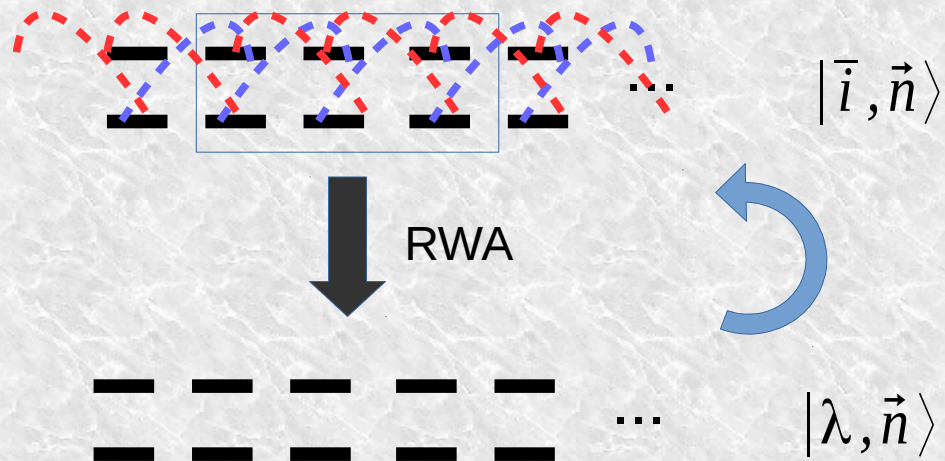
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Variational parametrization of the micromotion operator

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G. Torlai and R. G. Melko, PRL. **120**, 240503 (2018)

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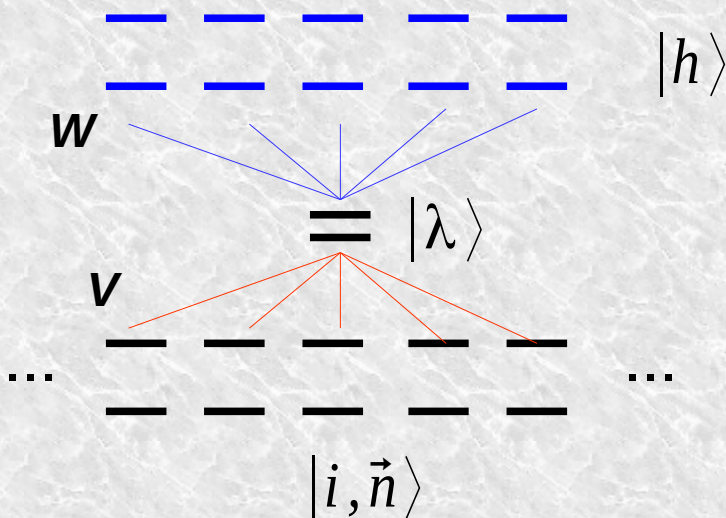
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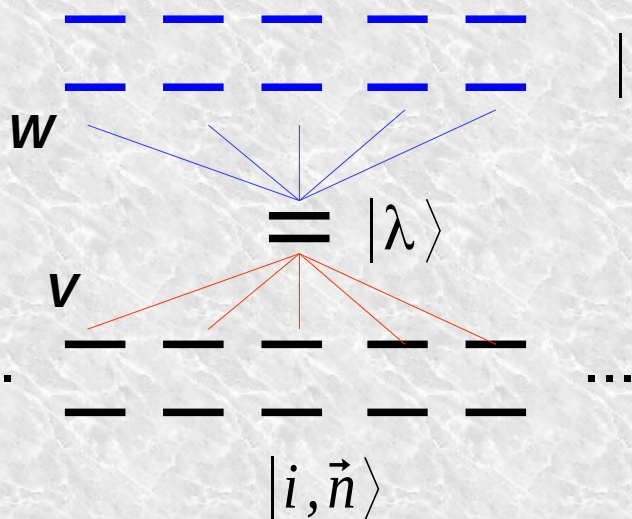
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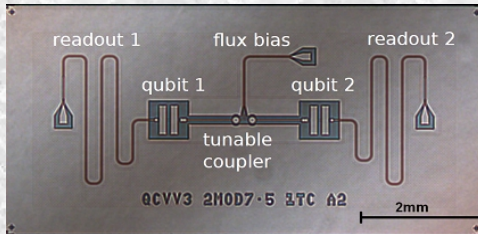
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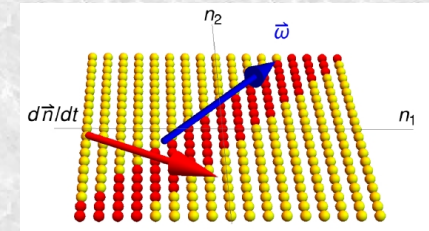
and a diagonal time-independent matrix.

Outlook

Quantum control



Time-domain quantum simulations

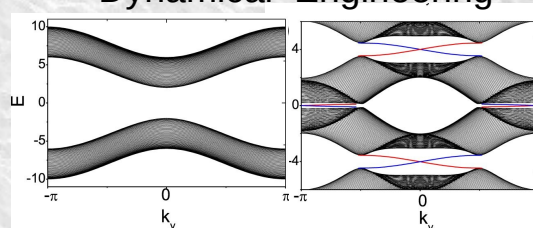


$$H = \sum_{\vec{n}} \hat{H}_{\vec{n}} e^{i\vec{n} \cdot \vec{\omega} t}$$

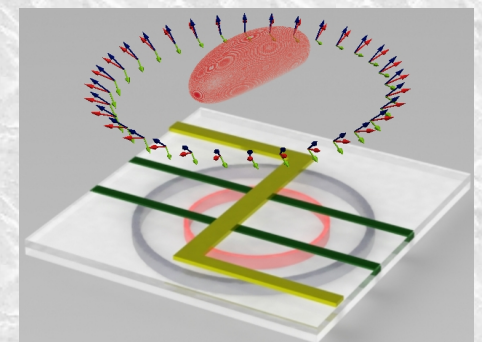
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Dynamical Engineering



Atomic Quantum technology



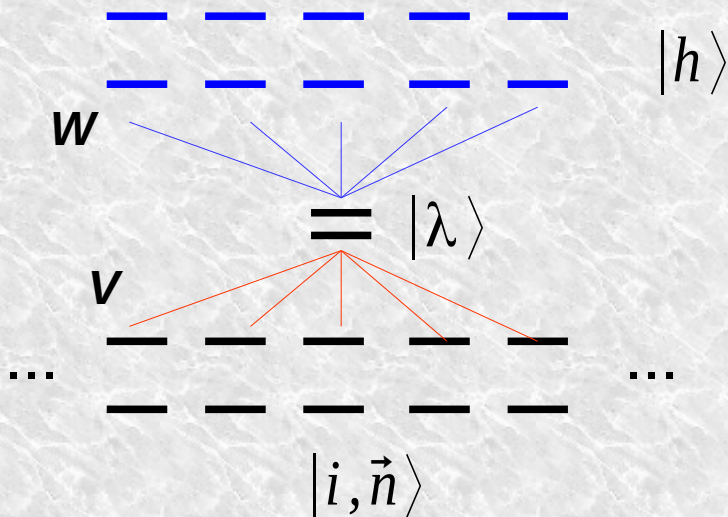
$U(t',t)$ with ML

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